Journal
of
The Franklin Institute
Devoted to Science and the Mechanic Arts

Vol. 236 AUGUST, 1943 No. 2

THE VARIATION OF MESOTRON INTENSITY WITH ALTITUDE AND LATITUDE, TOGETHER WITH ALLIED PHENOMENA, AND THE BEARING OF THESE MATTERS ON THE NATURE OF THE PRIMARY PARTICLES.

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PART II.

In Part I [1] we outlined the general purposes of the paper and summarized the conclusions reached. We also cited the contents of the various sections given here in detail in Part II. While these sections are self-explanatory, their meaning will be enhanced to the reader by a preliminary perusal of Part I.

SECTION I.

CALCULATION OF THE VERTICAL MESOTRON INTENSITY AT A DEPTH x BELOW A REGION OF PRODUCTION IN WHICH THE SPECTRAL ENERGY DISTRIBUTION OF THE MESOTRONS IS ASSIGNED.

Suppose that the contribution Δn_o to the vertical mesotron intensity at the altitude of production by the mesotrons whose energy lies between E and $E + dE$ is

$$\Delta n_o = F(E)dE. \qquad (1)$$

[1] JOUR. FRANK. INST., Vol. 236, No. 1, p. 1.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

Suppose that, as a result of ionization loss, $\bar{E}_x$ is the energy necessary for a mesotron to travel to the depth x below the altitude of production.

Let $(\Delta n)_x$ be the portion of Δn_o which survives at the depth x.

Provided that the value of E associated with the group Δn_o is such that $E > \bar{E}_x$, we have

$$-d(\Delta n)_x = \frac{(\Delta n)_x dx}{\beta_x c\tau_o(1-\beta_x^2)^{-\frac{1}{2}}} = \frac{m_o c^2(\Delta n)_x dx}{\beta_x c\tau_o[E - \bar{E}_x + m_o c^2]}, \qquad (2)$$

where m_o is the mesotron rest mass, c is the velocity of light, $\beta_x = v_x/c$, where v_x is the velocity at the depth x, and τ_o is the mean life for a stationary mesotron. In what follows, it will be sufficient, subject to subsequent justification in terms of the calculated results, to assume $\beta_x = 1$, except in the term $(1-\beta_x^2)^{-\frac{1}{2}}$.

If α is the energy loss per centimeter of path at sea-level, ρ_o the ratio of the density at the altitude of production to that at sea-level, and ρ the corresponding value for the depth x, we have

$$\bar{E}_x = \int_o^x \alpha\rho dx = \alpha\rho_o \int_o^x e^{\lambda x} dx = \frac{\alpha\rho_o}{\lambda}(e^{\lambda x} - 1), \qquad (3)$$

where λ is a constant defining the rate of increase of density with altitude.

Put

$$A \equiv E + m_o c^2 + \frac{\alpha\rho_o}{\lambda} \qquad (4)$$

and

$$y \equiv \frac{\alpha\rho_o}{\lambda} e^{\lambda x},$$

so that

$$\lambda dx = dy/y.$$

Then (2) becomes

$$\frac{-c\tau_o}{m_o c^2}\frac{d(\Delta n)_x}{(\Delta n)_x} = \frac{dy}{\lambda y(A-y)},$$

$$-\frac{\lambda c\tau_o}{m_o c^2}\log(\Delta n)_x = -\frac{1}{A}\left[\log\frac{A-y}{y} + \log B\right],$$

where B is a constant of integration. Thus

$$(\Delta n)_x = \left[B \left(\frac{A}{y} - 1 \right) \right]^{m_o c^2/c\tau_o \lambda A}. \qquad (5)$$

Thus, at $x = 0$, we have

$$F(E)dE = B^{m_o c^2/c\tau_o \lambda A} \left(\frac{E + m_o c^2}{\alpha \rho_o/\lambda} \right)^{m_o c^2/c\tau_o \lambda A}.$$

Thus

$$B^{m_o c^2/c\tau_o \lambda A} = \left[\frac{\alpha \rho_o}{\lambda (E + m_o c^2)} \right]^{m_o c^2/c\tau_o \lambda A} F(E)dE$$

and, from (5),

$$(\Delta n)_x = \left[\frac{\alpha \rho_o}{\lambda (E + m_o c^2)} \left(\frac{\lambda A e^{-\lambda x}}{\alpha \rho_o} - 1 \right) \right]^{m_o c^2/c\tau_o \lambda A} F(E)dE.$$

Inserting the value of A as defined by (4),

$$(\Delta n)_x = \left[\left\{ 1 + \frac{\alpha \rho_o}{\lambda (E + m_o c^2)} \right\} e^{-\lambda x} - \frac{\alpha \rho_o}{\lambda (E + m_o c^2)} \right]^{[\lambda c\tau_o (E/m_o c^2 + 1 + \alpha \rho_o/\lambda m_o c^2)]^{-1}} F(E)dE. \qquad (6)$$

This formula (6) represents the number supplied at the depth x by those mesotrons which, at the altitude of production, have energies between $(E - \bar{E}_x)$ and $(E - \bar{E}_x) + dE$. Equation (6) may be shortened by writing

$$G_o \equiv \alpha \rho_o/\lambda m_o c^2; \qquad G_x \equiv \alpha \rho_x/\lambda m_o c^2 \qquad \text{and} \qquad \gamma \equiv E/m_o c^2,$$

where ρ_x is the ratio of the density at the point defined by x to the density at sea-level.

With these substitutes, (6) reduces to

$$(\Delta n)_x = \left[1 + \left\{ \frac{G_o - G_x}{\gamma + 1} \right\} \right]^{[\lambda c\tau_o (\gamma + 1 + G_o)]^{-1}} \times \left[e^{-x/c\tau_o (\gamma + 1 + G_o)} \right] F(E)dE. \qquad (7)$$

By writing $x = x_o - h$, where x_o is the altitude of production, and h is the altitude at which the value of Δn_x is sought, both altitudes being expressed in relation to sea-level, our expression assumes a form in terms of the variable h, as

follows:

$$(\Delta n)_h = \left\{\left[1 + \left\{\frac{G_o - G_h}{\gamma + 1}\right\}\right]^{[\lambda c\tau_o(\gamma+1+G_o)]^{-1}}\right\} \times \{e^{-x_o/c\tau_o(\gamma+1+G_o)} e^{h/c\tau_o(\gamma+1+G_o)} F(E)dE.\}. \quad (8)$$

Naturally, and in general, in the case of (6), (7), and (8) n_x must be obtained by numerical integration.[2]

Simplification of (6) for the Case of High Altitudes of Mesotron Production. For high altitudes of mesotron production ρ_o is small and $\alpha\rho_o/\lambda(E + m_oc^2)$ is small compared with unity. Thus, in (6), this quantity may be neglected in the exponent and in the bracket where it is added to unity. It must not be neglected in the remaining place, however, since here it is not multiplied by the quantity $e^{-\lambda x}$ which is, in general, small.

Now if ρ_x is the ratio of the density at the point defined by x to the density at sea-level, $\rho_o = \rho_x e^{-\lambda x}$, so that, with the above approximations invoked, (6) becomes

$$(\Delta n)_x = \left[\left\{1 - \frac{\alpha\rho_x}{\lambda(E + m_oc^2)}\right\} e^{-\lambda x}\right]^{[\lambda c\tau_o(E/m_oc^2+1)]^{-1}} F(E)dE. \quad (9)$$

Let us now write

$$\frac{1}{\epsilon} \equiv \frac{E}{m_oc^2} + 1,$$

$$\mu_x \equiv \frac{x}{c\tau_o},$$

$$\nu_x \equiv \frac{\alpha\rho_x}{\lambda m_oc^2}.$$

[2] In integrating this equation over the range of E involved, we must observe that for any depth x we are concerned only with values of E (at $x = o$) greater than $\bar{E}_x$, the loss of energy over the range of o to x. In other words, our lower limit is

$$E = \bar{E}_x = \frac{\alpha\rho_o}{\lambda}(e^{\lambda x} - 1) = \frac{\alpha}{\lambda}(\rho_x - \rho_o)$$

unless the earth's magnetic field, through its action on the primaries, has imposed a higher limit. Thus, if E_{MP} is the limit for entry of the primaries, and if the primaries split into N mesotrons under the condition that the mesotrons are born at rest in the system of axes in which the primary moved, then the lower limit of the kinetic energy for the mesotrons will be E_{MP}/N. Strictly speaking, it will be even lower than this by $(1/N)$ times the energy lost by the primary by ioniza-

V.23
1943

Then (9) becomes

$$(\Delta n)_x = (1 - \nu_x\epsilon)^{\epsilon/\lambda c\tau_o} e^{-\mu_o x} F(E)dE. \tag{10}$$

If we apply this to the case where $F(E) = B/E^3$,

$$F(E)dE = K\frac{dE}{E^3} = -\frac{Kd\epsilon}{m_o^2c^4\epsilon^2}\left[\frac{1}{\epsilon} - 1\right]^{-3}$$

and (10) becomes

$$(\Delta n)_x = -\frac{K}{m_o^2c^4}(1 - \nu_x\epsilon)^{\epsilon/\lambda c\tau_o}\frac{e^{-\mu_o x}}{(1-\epsilon)^3}\epsilon d\epsilon$$

and

$$n_x = \frac{-K}{m_o^2c^4}\int_o^{\epsilon_o}\frac{(1 - \nu_x\epsilon)^{\epsilon/\lambda c\tau_o}}{(1-\epsilon)^3}e^{-\mu_o x}\epsilon d\epsilon, \tag{11}$$

where

$$\frac{1}{\epsilon_o} \equiv \frac{\bar{E}_x}{m_oc^2} + 1 \tag{12}$$

and, as shown in footnote (2), $\bar{E}_x$ is given by

$$\bar{E}_x = \frac{\alpha\rho_o}{\lambda}(e^{\lambda x} - 1) = \frac{\alpha}{\lambda}(\rho_x - \rho_o),$$

unless this value of $\bar{E}_x$ is less than the value imposed by the magnetic field through its action on the primaries in which case ϵ_o should be calculated from (12) by replacing $\bar{E}_x$ therein by E_{MP}/N, as shown in footnote (2).

The Spectral Energy Distribution at Any Altitude for the Case of Mesotron Production at a Fixed Altitude.

Let η be the water-equivalent depth from the top of the homogeneous atmosphere of the point at which the intensity is sought, and let ζ_o be the water-equivalent depth of the place of mesotron production. Then, if D is the actual density at a point

$$\eta - \zeta_o = \int_o^x Ddx = D_o\int_o^x e^{\lambda x}dx = \frac{D_o}{\lambda}(e^{\lambda x} - 1),$$

where D_o is the actual density at the place of mesotron production.

tion in descending to the level of production. This correction would be negligible for altitudes of production sufficiently high to have significance in the present theory, particularly on account of the factor $(1/N)$ in the correction.

We have also $D_o = \lambda\zeta_o$, so that

$$\frac{\eta}{\zeta_o} = e^{\lambda x}. \tag{13}$$

Again, if E_η is the energy at the depth η of a ray which had energy E at the place of mesotron production

$$E_\eta = E - \frac{\alpha}{D_a}(\eta - \zeta_o),$$

where D_a is the density of the atmosphere at sea-level.

Also, since

$$\rho_o = D_o/D_a = \frac{\lambda\zeta_o}{D_a}, \tag{14}$$

$$\frac{\alpha\rho_o}{\lambda(E+m_oc^2)} = \frac{\alpha\zeta_o}{D_a(E+m_oc^2)} = \frac{\alpha\zeta_o}{E_\eta D_a+\alpha(\eta-\zeta_o)+m_oc^2D_a}.$$

We are now in a position to substitute in (6)

$$(\Delta n)_\eta = \left[\frac{(E_\eta+m_oc^2)D_a\zeta_o/\eta}{E_\eta D_a+\alpha(\eta-\zeta_o)+m_oc^2D_a}\right]^{\left[\frac{m_oc^2D_a}{\lambda c\tau_o(E_\eta D_a+\alpha\eta+m_oc^2D_a)}\right]} \times F\left[E_\eta+\frac{\alpha}{D_a}(\eta-\zeta_o)\right]dE_\eta. \tag{15}$$

The coefficient of dE_η in this expression is, of course, the spectral energy distribution function at the point defined by η. Its lower limit is given by $E_\eta = 0$ unless E_{MP}/N is greater than the energy loss $\alpha(\eta - \zeta_o)/D_a$ from the layer of mesotron production to the point represented by η, in which case the lower limit should be $[E_{MP}/N - \alpha(\eta - \zeta_o)/D_a]$ in accordance with footnote (2).

SECTION II.

EXTENSION OF THE RESULTS OF SECTION I TO THE CASE WHERE MESOTRON PRODUCTION OCCURS AT ALL ALTITUDES.

If the rate of production of mesotrons is expressed only in a graphical form as a function of altitude, we can extend the calculations of Section I in an obvious manner by integrating graphically, for any altitude at which the mesotron intensity is desired, the contributions from all the altitudes of production above that altitude.

If the absorption of the primaries follows the simple exponential law, the desired results may be obtained simply, in algebraic form, as follows:

Let ζ refer to water equivalent distance from the top of the water equivalent atmosphere.

Suppose that $N_p f(E)dE$ is the intensity of the primaries, lying in the energy range E to $E + dE$, and that N_p is a function only of the water equivalent depth.[3] Suppose that

$$-\frac{d}{d\zeta}[N_p f(E)dE] = \sigma N_p f(E)dE,$$

so that

$$N_p = N_o e^{-\sigma\zeta},$$

where the absorption coefficient σ is concerned with mesotron production.

If $\Delta p_\zeta d\zeta$ is the number of mesotrons generated per second in the distance $d\zeta$ by the primaries lying in the range E to $E + dE$, and N is the number of mesotrons resulting from the disintegration of a single primary, we have

$$\Delta p_\zeta = -N\frac{dN_p}{d\zeta}[f(E)dE]d\zeta = N\sigma N_p[f(E)dE]d\zeta$$
$$= N\sigma N_o e^{-\sigma\zeta} f(E)dEd\zeta.$$

Thus if in equation (6), Section I, we replace $F(E)dE$ by $N\sigma N_o e^{-\sigma\zeta} f(E/N)d(E/N)d\zeta$ we obtain the contribution of $d\zeta$ to the mesotron intensity x cm. of actual distance below $d\zeta$.

Let us now express everything in terms of water equivalent distance.

In immediate analogy with (13), we have, for the altitude of production ζ, the relation $\text{Exp}[\lambda x] = \eta/\zeta$, and in immediate analogy with (14), we obtain

$$\rho_\zeta = \lambda\zeta/D_a,$$

[3] This implies that the probability of disintegration of a primary per unit distance of travel in the water equivalent atmosphere is independent of the energy, and that the energy distribution is not appreciably altered as a result of ionization energy loss by the protons in traveling the water equivalent distance. The justification of the latter assumption is given in footnote (2).

so that (6) yields

$$(\Delta n)_\eta = \left[\left\{1+\frac{\alpha\zeta}{D_a(E+m_o c^2)}\right\}\left\{\frac{\zeta}{\eta}\right\} - \frac{\alpha\zeta}{D_a(E+m_o c^2)}\right]^{[\lambda\sigma\tau_o(E/m_o c^2+1+\alpha\zeta/D_a m_o c^2)]^{-1}} + N\sigma N_o e^{-\sigma\zeta} f(E/N)d(E/N)d\zeta. \quad (16)$$

In integrating this expression to obtain the total intensity, we observe that for any given value of ζ, the lower limit for E is given by $\alpha(\eta-\zeta)D_a$ unless E_{MP}/N is greater than this quantity, in which case the lower limit for the said value of ζ is E_{MP}/N in accordance with footnote (2). Thus, having chosen η, the integration must first be made with regard to E for a fixed ζ, with the lower limit a function of ζ as above, and this integration must then be followed by an integration with respect to ζ, from $\zeta = 0$ to $\zeta \equiv \eta$. Naturally, these integrations must, in general, be carried out graphically.

V.23
1943

The Spectral Energy Distribution at Any Altitude for the Case of Mesotron Production at All Altitudes.

If in (16) we replace E by $E_\eta + \alpha(\eta-\zeta)/D_a$ we obtain

$$(\Delta n)_\eta = \left\{\left[\frac{(E_\eta+m_o c^2)D_a\zeta/\eta}{E_\eta D_a+\alpha(\eta-\zeta)+m_o c^2 D_a}\right]^{\left[\frac{m_o c^2 D_a}{\lambda\sigma\tau_o(E_\eta D_a+\alpha\eta+m_o c^2 D_a)}\right]}\right\} \times\left\{N\sigma N_o e^{-\sigma\zeta} f\left[\frac{E_\eta}{N}+\frac{\alpha}{D_a}(\eta-\zeta)\right]\right\} d\frac{E_\eta}{N}d\zeta. \quad (17)$$

The coefficient of dE_η in this expression is, of course, the part of the spectral energy distribution function at the point defined by η and contributed by the range $d\zeta$. The procedure for using (17) is as follows:

If $[E_{MP}/N - \alpha\eta/D_a]$ is greater than zero, let its value be $E_{\zeta 1}$. Then $E_{\zeta 1}$ represents the absolute lower limit for E_ζ below which no mesotrons are to be found. For this value of $E_{\zeta 1}$ our lower limit of integration for η is zero, and the upper limit is infinitesimally different from it, so that the distribution function is zero at $E_{\zeta 1}$. However, if we assign a value $E_{\zeta 2}$ greater than $E_{\zeta 1}$, the lower limit of integration

for ζ is zero, and the upper limit is given by ζ_2, where

$$E_{MP}/N - \alpha(\eta - \zeta_2)/D_a = E_{\zeta_2}, \tag{18}$$

unless the value of ζ_2 so calculated is greater than η, in which case the upper limit is η.

If $[E_{MP}/N - \alpha\eta/D_a]$ is zero, then zero is the lower limit for the integration with regard to ζ and the upper limit is infinitesimally different from it. If, for this case, we consider a value E_{ζ_3} greater than zero, the lower limit of integration for ζ is zero and the upper limit is given by ζ_3, where

$$E_{MP}/N - \alpha(\eta - \zeta_3)/D_a = E_{\zeta_3}, \tag{19}$$

unless the value of ζ_3 so calculated is greater than η, in which case the upper limit is η.

If $[E_{MP}/N - \alpha\eta/D_a]$ is less than zero, then the lower limit of E_ζ is zero. The lower limit for the ζ integration is then given by ζ_4, where

$$E_{MP}/N - \alpha(\eta - \zeta_4)/D_a = 0, \tag{20}$$

and the upper limit is infinitesimally different from it. If, however, we consider a value E_{ζ_4} greater than zero, the lower limit of the ζ integration is given by ζ_4, and the upper limit by ζ_5, where

$$E_{MP}/N - \alpha(\eta - \zeta_5)/D_a = E_{\zeta_5}, \tag{21}$$

unless the value of ζ_5 is greater than η, in which case the upper limit is η.

SECTION III.

APPLICATION OF EXPERIMENTAL DATA TO THE RESULTS OF SECTION I.

Determination of the Lower Limit of Energy Spectrum of Mesotrons and Altitude of Production, and the Determination of the Mass of the Primary.

1. *General Considerations.* On integrating (8) with regard to $\gamma(=E/m_o c^2)$ for the range of γ concerned, and for an assigned altitude of mesotron production x_o, we obtain the vertical intensity at the altitude h. The expression contains three unassigned constants, a constant, which we may call K, occurring as a multiplicative factor in $F(E)$, the altitude of production x_o, and the lower limit of integration for γ. The constant K is concerned only in fixing the absolute intensity at any altitude as distinct from the intensity relative to some fixed altitude—sea-level, for example. While in practice it

is convenient to deal with intensities relative to sea-level intensity and thus eliminate K from the start, the discussion of the rather subtle considerations to follow can be made rather more easily in terms of the absolute intensities, so that we shall retain K.

2. If we consider three different altitudes, h_o, h_1, and h_2 of increasing amount, we may write down the theoretical expressions for the intensity at each altitude and equate it to the experimental value [4] for that altitude. If γ_o, γ_1, and γ_2 are the corresponding lower limits for *integration* for γ, and if γ_m is the absolute lower limit of γ in the expression of $F(E)$,[5] then, in accordance with footnote (2)

$$\gamma_o = \gamma_1 = \gamma_2 = \gamma_m \qquad \text{if} \qquad m_o c^2 \gamma_m > \frac{\alpha}{\lambda}(\rho_{ho} - \rho_o), \quad (22)$$

$$m_o c^2 \gamma_o = \frac{\alpha}{\lambda}(\rho_{ho} - \rho_o); \qquad \gamma_1 = \gamma_2 = \gamma_m$$

$$\text{if} \qquad \frac{\alpha}{\lambda}(\rho_{ho} - \rho_o) > m_o c^2 \gamma_m > \frac{\alpha}{\lambda}(\rho_{h1} - \rho_o), \quad (23)$$

$$m_o c^2 \gamma_o = \frac{\alpha}{\lambda}(\rho_{ho} - \rho_o); \quad m_o c^2 \gamma_1 = \frac{\alpha}{\lambda}(\rho_{h1} - \rho_o); \quad \gamma_2 = \gamma_m$$

$$\text{if} \qquad \frac{\alpha}{\lambda}(\rho_{h1} - \rho_o) > m_o c^2 \gamma_m > \frac{\alpha}{\lambda}(\rho_{h2} - \rho_o), \quad (24)$$

$$m_o c^2 \gamma_o = \frac{\alpha}{\lambda}(\rho_{ho} - \rho_o); \quad m_o c^2 \gamma_1 = \frac{\alpha}{\lambda}(\rho_{h1} - \rho_o);$$

$$m_o c^2 \gamma_2 = \frac{\alpha}{\lambda}(\rho_{h2} - \rho_o)$$

$$\text{if} \qquad \frac{\alpha}{\lambda}(\rho_{h2} - \rho_o) > m_o c^2 \gamma_m. \quad (25)$$

In the last case, which corresponds to the case where γ_m is too small to permit penetration of the corresponding mesotrons down even to the highest altitude h_2, the three equations do not involve γ_m and so cannot serve to determine it. In this case, two of the equations would suffice to determine x_o and

[4] M. Schein, W. P. Jesse, and E. O. Wollan, *Phys. Rev.*, **59**, 615 (1941).

[5] This limit is determined, of course, ultimately from the lower limit for the primaries which in turn is determined by the magnetic field. However, it is not necessary at this stage of the discussion to recognize more than the fact that there *is* a lower limit for $F(E)$ at the altitude x_o, regardless of the origin of that lower limit.

K, and the problem of satisfying the third would be the same as that of satisfying the equation for any altitude within the range h_o to h_2. If such equations are automatically satisfied with the constants determined by two of them, the theory is correct; if they are not so satisfied, the theory is incorrect.

If any of the cases (22) to (25) applied, and we knew this beforehand, we could use our three intensity equations to determine K, x_o and γ_m. In practice, however, we do not know beforehand which of the conditions (22) to (25) holds, since we do not know γ_m. What then must be our procedure?

3. Suppose that we arbitrarily assign the limit γ_m to all three equations, and then use the equations to determine it. If, with the γ_m so determined, (22) is satisfied, we have our complete solution for K, x_o and γ_m.

What will happen, however, if the condition is really that represented by (22) or (23), although we do not know it and we have arbitrarily assigned the lower limit as γ_m for all three equations? Nothing can prevent us from solving the equations and determining a γ_m, but since, by hypothesis, the condition (22) is not satisfied, our γ_m will satisfy the condition

$$m_o c^2 \gamma_m < \frac{\alpha}{\lambda}(\rho_{h_o} - \rho_o). \tag{26}$$

Our equation for the altitude h_o will then have a curious meaning, a meaning possible mathematically, but impossible physically. Some of the mesotrons represented by the integration of (8) as contributions at h_o will, at h_o and for some distance above, have negative energy. This is perfectly possible mathematically. Mathematically, a mesotron can go on ionizing until its energy is zero, and then continue to ionize at the expense of its credit balance in its "bank" of energy, which credit balance is now zero and has to become negative to allow the impoverished mesotron to continue its operations. But while the mathematics is satisfied by such a discreditable procedure on the part of the mesotron, the physicist is not satisfied and denies its possibility.[6] Thus if,

[6] Of course, when we recognize a fact not evident in the intensity expression itself, the fact that at any altitude h

$$E_h/m_o c^2 = [(1 - \beta_h^2)^{-\frac{1}{2}} - 1],$$

we realize that if E_h were negative $(1 - \beta_h^2)^{-\frac{1}{2}}$ would be less than unity, so that $1 - \beta_h^2$ would be greater than unity, a result impossible for real values of β_h.

having assumed γ_m as the limit for all three equations, and having solved for K, x_o, and γ_m, we find that (26) holds, we must neglect the solution and try again, assuming the limits given by (23). If the γ_m so determined satisfies (23), our solution is satisfactory. On the other hand, if $m_o c^2 \gamma_m < \alpha(\rho_{h1} - \rho_o)/\lambda$, we must try the limits assigned by (24). If the γ_m so determined satisfies (24), we have our solution. If it does not, our last possibility of determining γ_m from the observations at h_o, h_1, and h_2 has evaporated, and we are left with the limits cited by (25) which, when applied in the equations, serve to determine from two altitudes only the constants K and x_o.

4. Of course, the whole point involved in the above is to the effect that if the true lower limit γ_m corresponds to an energy less than that necessary for a mesotron to enable it to reach down to the highest altitude used in our calculations, those calculations cannot, in the nature of things, give any information with regard to γ_m. We cannot test the matter before making the calculations because we do not wish to calculate γ_m on the basis of the formation of mesotrons from some *assigned* primary particle whose energy for entry through the magnetic field is fixed by its mass and charge. We wish rather to determine γ_m from our observations and then work back to the primary particle.

5. *Practical Method of Procedure.* In view of the fact that, in general, the integration with respect to γ cannot actually be carried out, we are not provided with an integrated equation involving γ_m, even when we choose γ_m as a limit of integration, and in practice, therefore, it is necessary to proceed in a manner different from that outlined above, a manner, in fact, such as the following:

Even if the purely analytical method were possible of application, we should at least have to assume a form for $F(E)$. Consequently, we assume the form

$$F(E)dE = KdE/E^3 = (K/m_o^2c^4)d\gamma/\gamma^3.$$

We may then plot against γ the coefficient of $d\gamma$ in (8) for the three altitudes h_o, h_1, h_2, and for an assigned value of x_o. Corresponding curves may be drawn for other values of x_o. Considering the curves for any given value of x_o, what-

V.23
1943

ever lower limit we assign for γ, we can now determine the desired integral from that limit to infinity by measuring the corresponding area.

We now try whether the lower limit can be the same for all three altitudes. We try whether, in fact, the conditions are those represented by (22). For any assigned x_o, we plot against the lower limit, say γ', the intensity ratio I_{h_1}/I_{h_o} for the altitudes h_1 and h_o. We find the point on this curve for which the ratio is equal to the experimental value. Let γ'' be the value of γ' for this point. We now choose another value of x_o and proceed as before. In this way we are able to plot a curve of γ'' against x_o characterized by the condition that for every pair of values of γ'' and x_o on that curve, I_{h_1}/I_{h_o} has the correct experimental value. We now repeat the whole process for the altitudes h_o and h_2, and plot a new curve of γ'' against x_o on the same graph. At the value of γ'', where the curves cross, we have a value of γ'' and of x_o which give the correct values for I_{h_1}/I_{h_o} and I_{h_2}/I_{h_o}. If the value of γ'' so determined satisfies the condition imposed on γ_m by (22), then this value of γ'' *is* γ_m.

If the value found for γ'' as above does not satisfy (22), we must proceed as before, except that for γ_o we impose the definite limit given by (23) and plot I_{h_1}/I_{h_o} against γ' for an assigned value of x_o, with the limit γ' used only in the integral for the altitude h_1. We then determine a value γ'' to yield the experimental ratio for I_{h_1}/I_{h_o}. We repeat the process for different values of x_o and plot γ'' against x_o. Following this we now do a similar thing for the ratio I_{h_2}/I_{h_o}, and plot another curve of γ'' against x_o. If the value of γ'' corresponding to the intersection satisfies the condition imposed by (23) on γ_m, it *is* γ_m. If it does not satisfy this condition we must proceed in analogous manner with the limits for altitudes h_o and h_1 prescribed by (24). If again we are unsuccessful, there remains only the assignment of limits imposed by (25).

Theory of Determination of Mass of the Primary. If we have determined γ_m as above, we may get information about the primary particle as follows:

6. Remembering that it is the minimum *momentum* for entry which is determined by the magnetic field, if m is the

mass of the primary particle, and ne its charge, e being the electronic charge, we then have

$$\frac{m}{n}\frac{\beta_m}{(1-\beta_m^2)^{1/2}} = ef(\varphi), \tag{27}$$

where β_m is the lower limit for entry of the particle, and $f(\varphi)$ is a function of the magnetic latitude φ involving nothing but terrestrial magnetic data, and independent of the nature of the particle.

If we have determined γ_m for the mesotrons by procedures such as those outlined above, and recognizing that, in line with the assumption governing the splitting of heavy particles into mesotrons, the β for a given mesotron is the same as that of its parent heavy particle, we have

$$(1-\beta_m^2)^{-1/2} = \gamma_m + 1. \tag{28}$$

This value of β_m, when substituted in (27), gives m/n; and if we are willing to assume that the primary particle has a single electronic charge, we have the value of m.

7. It is clear that for the case of protons, for example, where $m = 10\ m_o$, and for which the minimum energy of vertical entry at magnetic latitude 50° is 2.2×10^9 e.v., we should have $2.2 \times 10^9/10$ for the minimum mesotron energy. This would not permit a mesotron with that energy to travel down as far as the top of Pike's Peak and the heavier the primary particle, the smaller the value of β_m, as may be seen from (27). It is not possible to use observations extending to altitudes which are in any way comparable with x_o, since the imperfections of the assumption of a single altitude of production become enhanced. For this reason, we have not used observations for altitudes above 4 kilometers, and it turns out that from such data we are limited to the condition (25), so that our equations do not yield a value for γ_m. If we had observations on increase of mesotron intensity with altitude at the magnetic equator, protons would yield a minimum mesotron energy of $14 \times 10^9/10 = 1.4 \times 10^9$ e.v., which would just permit the corresponding γ_m to be the lower limit of integration for all values of h down to sea-level since the energy to penetrate the whole atmosphere is 1.4×10^9 e.v. If intensity altitude measurements existed for mesotrons at the

equator, we could use these observations for the purpose of determining γ_m.

8. *Determination of Altitude of Production from Intensity-Altitude Data of Schein, Jesse, and Wollan.* However, by using the condition (25) for the limits of integration in the latitude of the measurements of Schein, Jesse, and Wollan, we can determined x_0, the altitude of production. It is convenient to express matters by plotting the intensity ratio for altitudes 4 kilometers and sea-level against the altitude of production.[7] Fig. 1 represents the results.[8] The intensity

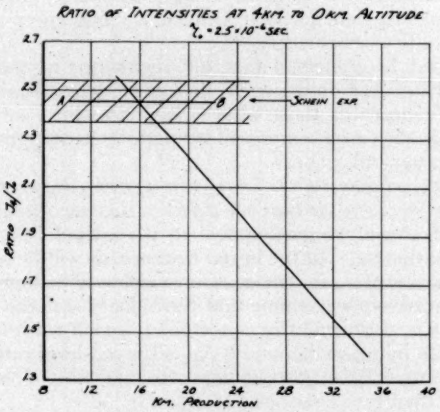


FIG. 1

ratio yielded by the experimental data is indicated, with the limit of uncertainty indicated by the shaded region, by the line AB, which yields the value 16 kilometers for the altitude of production x_0. At first sight it might seem that the value 16 kilometers for the altitude of production is low in the light of our experimental knowledge of the variation of mesotron intensity with altitude at high altitudes. However, it is to be

[7] We do this by the graphical method already outlined, the lower limits of integragion being those expressed by (25).

[8] The constants assumed are $\tau_0 = 2.5 \times 10^{-6}$ sec.; $\alpha = 1800$ e.v. per cm. at atmospheric pressure; $\lambda = (1/8) \times 10^5$; $m_0c^2 = 0.85 \times 10^8$ e.v.

remembered that this value is a very special sort of average. It is lower than the true average altitude of production; for if we fix our attention on the true average altitude of production at, let us say, H kilometers, and if we should mark off, above and below it, corresponding strata which make equal contributions to the production of vertically directed mesotrons, the upper strata would be increasingly farther away from the H kilometer altitude and their contributions to the mesotron intensity at sea-level would be correspondingly smaller. In other words, observations at low altitudes, when interpreted in terms of a single altitude of production, would tend to yield for that altitude a value lower than the true altitude of production.

It must be confessed that all discussions in terms of a single altitude of production are to some extent unsatisfactory for such reasons as those above cited, and in a subsequent communication an attempt will be made to handle the matter in a more general manner.

9. *Determination of the Lower Limit of the Mesotron Energy Spectrum by Use of the Latitude Effect.* Having obtained x_o as described above, we may appeal to the latitude effect for a determination γ_m. In the initial instance, as will be seen, it is γ_{m_E}, the value of γ_m for the equator which will be determined.

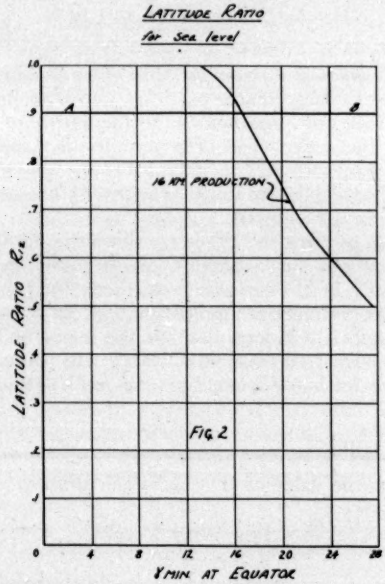
If tentatively we assume that the value γ_{m_E} at the equator is such as to permit all the mesotrons to reach sea-level apart from death by mean life, and if R_{12} is the sea-level ratio of the vertical intensity at the equator to that at 50° magnetic latitude, then

$$R_{12} = \frac{\int_{\gamma_{m_E}}^{\infty} \psi(\gamma, x_o) d\gamma}{\int_{\frac{\alpha(1-\rho_x)}{m_o c^2 \lambda}}^{\infty} \psi(\gamma, x_o) d\gamma}, \qquad (29)$$

where $\psi(\gamma, x_o)$ is the coefficient of $d\gamma$ in (8) after $F(E)dE$ has been replaced by $(K/m_o^2c^4)d\gamma/\gamma^3$. It is to be noted that $\rho_x = 1$ at sea-level.

Now utilizing the value $x_o = 16$ kilometers determined as described above, it is possible to plot R_{12} against the lower limit of the upper integral in (29). Fig. 2 shows the result.

Now the experimental value of R_{12} is represented by the ordinate of the line AB.[9] We find consequently that $\gamma_{mE} = 16.6$. This corresponds to an energy $0.85 \times 10^8 \times 16.6 = 1.4 \times 10^9$



e.v. for the lower limit of energy for the mesotrons, and this justifies the procedure involved in supposing that the proper lower limit in the numerator of (29) is γ_m rather than

[9] Strictly speaking we should use the latitude ratio for vertical rays and for mesotrons only. T. H. Johnson and D. N. Read (*Phys Rev.*, **51**, 560 (1937)) record a vertical latitude ratio of 12 per cent. with some reservations. A. H. Compton's results from a world survey give 10.3 per cent., but Compton gives reasons for supposing that part of this is due to a temperature effect and that only 7.2 per cent. remains as a magnetic latitude effect. This result is, however, for rays from all directions, and would be expected to be less than for vertical rays. We have assumed a latitude effect of 10 per cent. as a value probably near the truth for vertical mesotrons. More data are needed on this important matter.

$\alpha(1 - \rho_0)/m_0c^2\lambda$. Indeed the very existence of a latitude ratio different from unity guarantees this from the start. The above value of γ_{mE} corresponds to a value of β_m given by

$$(1 - \beta_m^2)^{-1/2} = \gamma_{mE} + 1 = 17.6.$$

This gives $\beta_m = 1 - 0.0015$.

10. *The Numerical Value of the Mass of the Primary Particle.* Turning now to the primary particles, and realizing that, in accordance with our hypothesis as to the nature of mesotron production, the value of β of a primary is handed on to the offspring mesotrons, we may take the above value of $(1 - \beta_m^2)^{-1/2}$ as applicable to the primaries. The corresponding value of β_m is $1 - 0.001$.

Now $ef(\psi)$ as cited in (27) is $1/c$ times the minimum momentum necessary for a particle of electronic charge e to enter vertically at the latitude concerned—in this case zero degrees. This minimum momentum for entry, unlike the energy, is explicitly independent of the mass, and depends only upon terrestrial magnetic data. Its value for the magnetic equator is 8.0×10^{-13} gm. cm./sec. Thus,

$$\frac{m}{n}\frac{\beta_m}{(1 - \beta_m^2)^{1/2}} = 8.0 \times 10^{-13}/3 \times 10^{10} = 2.7 \times 10^{-23},$$

so that, using the value of $(1 - \beta_m^2)^{1/2}$ cited above,

$$\frac{m}{n} \times 17.6 = 2.7 \times 10^{-23}.$$

If n = unity as corresponding to a single electronic charge, $m = 1.53 \times 10^{-24}$ gm. and corresponds to the mass 1.6×10^{-24} gm. of the proton within the limits of experimental error.[10]

Comparison Between Theoretical and Experimental Data on Spectral Energy Distribution of Mesotrons at Sea-Level.

As already stated, the spectral energy distribution for vertical rays is given by the coefficient of dE_η in (15). For sea-level, η is the water equivalent thickness of the atmos-

[10] In a recent publication, *Phys. Rev.*, 63, 210 (1943), the writer has based this conclusion on the ratio of increase of latitude effect with altitude. This method is not as satisfactory as that here given on account of the uncertainty of the data. The calculations here given are based, moreover, on data more satisfactory in other respects.

phere, so that, assuming an inverse cube form for F, the spectral energy distribution for vertical mesotrons at sea-level is given by $F(E_\eta)$, where

$$F(E_\eta) = K\left[\frac{(E_\eta + m_o c^2)D_a\zeta_o/\eta_a}{E_\eta D_a + \alpha(\eta_a - \zeta_o) + m_o c^2 D_a}\right]^{\left[\frac{m_o c^3 D_a}{\lambda c\tau_o(E_\eta D_a + \alpha\eta_a + m_o c^3 D_a)}\right]} \times\left[E_\eta + \frac{\alpha}{D_a}(\eta_a - \zeta_o)\right]^{-3}, \quad (30)$$

where K is a constant, and the lower limit for E_η is zero for a latitude of 50°. The quantity ζ_o is the water equivalent depth of the 16 kilometer altitude of production already determined. The other relevant magnitudes are defined in footnote (8).

The experimental data which we shall use for comparison are those of P. M. S. Blackett and those of H. Jones.[11] However, since the data were obtained by a counter control which did not confine the observations to strictly vertical rays, we cannot use (30) directly. While the dimensions of Blackett's control counters are not specified, it is probable that the arrangement is somewhat similar to that used by H. Jones, in which the rays examined were those confined to a plane and to an angle in that plane extending about 45 degrees on each side of the vertical.

In amending (30) to apply to any zenith angle θ, we must first determine for that θ a new altitude of production x_1, defined in relation to the altitude of production x_o for vertical rays by the condition

$$\int_{x_o}^{\infty} e^{-\lambda x}dx = \int_{x_1}^{\infty} e^{-\lambda x}dx/\cos\theta.$$

This leads to

$$x_1 - x_o = (1/\lambda)\log\cos\theta$$

and the water equivalent of the distance $x_1 - x_o$ is $\bar{D}_o/\cos\theta$, where $\bar{D}_o$ is the mean density of the atmosphere between x_o and x_1. The remaining changes in (30), necessary to make it apply to any angle θ, are easily seen to be the incorporation of

[11] P. M. S. Blackett, *Proc. Roy. Soc.*, **A159**, 1 (1937); H. Jones, *Rev. Mod. Phys.*, **11**, 235 (1939).

$\cos\theta$ as a factor to λ and the division of the term $\alpha(\eta - \zeta_0)$ by $\cos\theta$ in the two places where it occurs, and the term $\alpha\eta$ by $\cos\theta$ in the place where it occurs.

Figure 3 shows a comparison of the theoretical with the experimental data, based upon the value of x_0 corresponding to 16 kilometers. The squares represent the results of Blackett,

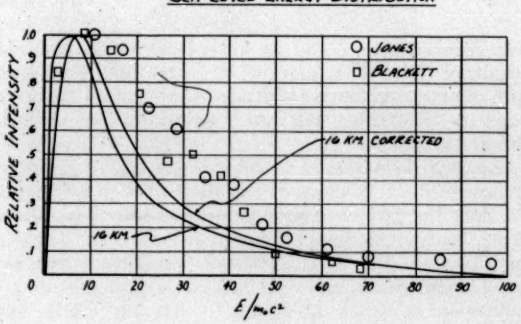


FIG. 3

and the circles those of Jones. The upper of the two full curves takes into account the angular distribution as above, and the lower curve is based upon strictly vertical rays. It cannot be claimed that the agreement is very good. A higher altitude of production would fit the experimental data better, but such an assumption would not be in harmony with the conclusions reached earlier as to the altitude of production. It is probable that a large part of the discrepancy is attributable to the assumption of a *single* altitude of production. Apart from these defects, however, and with better data and a more complete theory along the same general lines, it is worthy of notice that the sea-level energy distribution curve really provides a method of assigning the energy distribution at high altitudes, here assumed of the inverse cube form.[12]

[12] As this paper goes to press, new calculations of the energy distribution, and of other matters relevant to this paper have been made for the case of mesotron production at all altitudes instead of at a single altitude. The harmonization of theory with the experimental data becomes much enhanced by this procedure.

V.23
1943

SECTION IV.

THE ANGULAR DISTRIBUTIONS OF ELECTRONS EJECTED DURING THE DEATH OF MESOTRONS AND CONCLUSIONS AS TO THE NATURE OF A PART OF THE INCOMING PRIMARY RADIATION.

We may define the problem as one in which we have a beam of electrons of assigned spectral energy distribution and intensity traveling in a definite direction, and we wish to determine the intensity at any angle to that direction and, as a corollary to that result, the average angle of deviation from that direction. Our calculations will be made on the hypothesis that the electrons are emitted, on the average, symmetrically in all directions in the frame of reference of a group of mesotrons traveling with equal velocity.

For the purposes of record and subsequent possible use, we shall attack the problem rather more generally than would be necessary for the minimum requirements of this paper.

General Expression for the Average Deviation. Let the spectral energy distribution for the mesotrons be defined by

$$\delta n = F(E)dE.$$

If Θ is the average angle of deviation above referred to,

$$\Theta \int \delta n = \Sigma_2 \Sigma_1 \theta_E,$$

where Σ_1 refers to all of the individual electrons in a range dE, and Σ_2 refers to a summation for all the elements of range dE.

Now, if $\bar{\theta}_E$ is the average deviation for an electron of energy E, $\Sigma_1 \theta_E = \bar{\theta}_E F(E)dE$ and

$$\Sigma_2 \Sigma_1 \theta_E = \int \bar{\theta}_E F(E)dE.$$

Thus

$$\Theta = \frac{\int \bar{\theta}_E F(E)dE}{\int F(E)dE}; \qquad (31)$$

the upper limit of the integrals is infinity and the lower limit is given, on the line of the views already developed, by E_M/N, where E_M is the limit for entry of the primary particle through the magnetic field for the direction concerned, and N is the number of mesotrons born from a single primary particle.

We shall be able to make use of this expression after we have developed the expression for θ_E.

Determination of $\bar{\theta}_E$, the Average Angle of Deviation of the Electrons from the Original Mesotron Direction for a Given Mesotron Energy E. Let the mesotron move with velocity $v(=\beta c)$ parallel to the axis of x in the stationary system S. Let u_x and u_y be, respectively, the components of electron velocity parallel to and perpendicular to the x direction. Let u_x' and u_y' represent corresponding quantities in the system S' which moves with the mesotron. We have

$$u_x = \frac{u_x' + v}{1 + u_x'v/c^2}; \qquad u_y = \frac{(1-\beta^2)^{1/2}u_y'}{1 + u_x'v/c^2}.$$

If θ is the angle of deflection of the electron from the original mesotron path

$$\tan\theta = \frac{(1-\beta^2)^{1/2}u_y'}{u_y' + v} = \frac{(1-\beta^2)^{1/2}\tan\theta}{1 + v/\omega'\cos\theta'},$$

where ω' is the velocity of electron emission measured in S'. Thus

$$\frac{\tan\theta}{(1-\beta^2)^{1/2}} = \frac{\tan\theta'}{1 + \beta c/\omega'\cos\theta'} = \frac{\sin\theta'}{\cos\theta' + \beta c/\omega'}.$$

Put $c/\omega' = \alpha$.[13] Then

$$\frac{\tan^2\theta}{(1-\beta^2)} = \frac{\sin^2\theta'}{(\cos\theta' + \alpha\beta)^2} = \frac{1-\cos^2\theta'}{(\cos\theta' + \alpha\beta)^2}. \qquad (32)$$

Let us write

$$x \equiv \cos\theta; \qquad x' \equiv \cos\theta'.$$

Then (32) yields

$$\frac{1-x'^2}{(x' + \alpha\beta)^2} = \frac{1-x^2}{x^2(1-\beta^2)}, \qquad (33)$$

from which we derive

$$\frac{dx'}{dx} = \frac{(x' + \alpha\beta)^3}{(1-\beta^2)x^3(1 + \alpha\beta x')}. \qquad (34)$$

[13] Naturally this α, adopted here for a temporary purpose, has no connection with the α of Section I. A similar remark applies to the quantities x and x' used in equation (33).

Now theoretically, we could solve (33) for x' as a function of x and write (34) in the form

$$\frac{dx'}{dx} = \varphi(x), \tag{35}$$

where $\varphi(x)$ is a function of x, α, and β, which can theoretically be calculated and is given by the right-hand side of (34), so that

$$\varphi(x) = \left[\frac{(x' + \alpha\beta)^3}{(1 - \beta^2)(1 + \alpha\beta x')x^3}\right]_x,$$

the subscript intending to imply that the quantities in the square bracket are to be expressed as functions of x, by solving (33).

Now if I' is the electron emission per unit solid angle from a group of similarly directed mesotrons of equal energy, the number of electrons emitted within the limits prescribed by θ' and $\theta' + d\theta'$ is dn', where

$$dn' = 2\pi I' \sin\theta' d\theta' = -2\pi I' d(\cos\theta') = -2\pi I' dx'.$$

Thus

$$dn' = -2\pi I' \frac{dx'}{dx} dx = -2\pi I' \varphi(x) dx.$$

Since $-dx = \sin\theta d\theta$, this tells us that the intensity I in S corresponding to I' in S' is given by

$$I = I'\varphi(x). \tag{36}$$

Average Angular Deviation of the Electrons from the Original Direction of a Mesotron Beam. The formal procedure is to solve (33) for x' as a function of x, as already stated, so that $\varphi(x)$ may be obtained as an explicit function of x. Then, since $x = \cos\theta$, equation (36) leads to

$$dn = 2\pi I' \varphi(E, \cos\theta) \sin\theta d\theta \tag{37}$$

for the number dn of electrons ejected within the range θ to $\theta + d\theta$, where we have inserted an E with $\cos\theta$ in the function φ, to call attention to the fact that the function applies to a definite energy E and involves that energy in its form.[14]

[14] Explicitly φ involves β and α, but β is defined by E, and α, which, for practical purposes, is equal to unity, is formally equal to c/ω', where ω' is determined as a function only of the rest mass energy of the mesotron.

Then the value of $\bar{\theta}_E$ necessary to make (31) yield the desired quantity Θ is

$$\bar{\theta}_E = \frac{\int_o^\pi 2\pi I' \theta \varphi(E, \cos\theta) \sin\theta d\theta}{\int_o^\pi 2\pi I' \varphi(E, \cos\theta) \sin\theta d\theta}. \tag{38}$$

In general, the foregoing method for evaluating $\bar{\theta}_E$ is too complicated. A more practical method is the following: We plot x as a function of x', for an assigned value of β, and so of E, using (33). It is then possible to read off the value of x' for any assigned value of x, and so, from (34), to plot dx'/dx as a function of x. Since, as may be recalled from (35), $\varphi(x)$ is equal to dx'/dx, we may now plot $\varphi(x)$ as a function of x, and remembering that $x = \cos\theta$, we may use (38) to obtain $\bar{\theta}_E$ for any value of E. Equation (31) may then be used to evaluate Θ graphically.

V.23
1943

Application to a Comparison of the Horizontal and Vertical Intensities.

In order properly to take account of the implications of the foregoing calculations in their effect upon the ratio of the horizontal to the vertical intensity, it is necessary to take account of the radiation incident from all angles, and as part of the whole story, the law of absorption of the primaries becomes involved. The only practical plan is to proceed on the basis of some simple and plausible assumption in this matter, and we shall assume that a beam of primaries of N rays is absorbed according to a law given by

$$\frac{-dN}{dp} = \frac{\sigma N}{\cos\theta}, \tag{39}$$

where $-dN$ is the diminution in number in the element of path during which the pressure increases by dp, and θ is the zenith angle of the beam. We shall assume σ to be independent of the energy of the rays. On writing $x = \cos\theta$, and N_o for the value of N at the limit of the atmosphere, we have

$$N = N_o e^{-\sigma p/x}. \tag{40}$$

The number of these rays which die in the path length corresponding to the pressure change dp is $(\sigma/x)N_o \exp[-\sigma p/x]dp$, and if k is the number of mesotrons produced by the death of one primary, the number of mesotrons produced in the aforesaid path length by the death of the primaries is $(\sigma k/x)N_o \exp[-\sigma p/x]dp$.

The contribution which these mesotrons make to the mesotron intensity at a lower altitude, where the pressure is p_o, is ΔM_o, where

$$\Delta M_o = (\sigma k/x)N_o e^{-\sigma p/x} e^{-\zeta/\beta c\tau_o \epsilon} dp, \tag{41}$$

where τ_o is the mean life of a mesotron at rest, $\epsilon = (1 - \beta^2)^{-1/2}$, and ζ is the distance of the point where the pressure is p_o from the point where it is p, the distance being measured along a line inclined at a zenith angle θ.[15] Now since p is of the form

$$p = p_o e^{-\lambda\zeta \cos\theta} = p_o e^{-\lambda\zeta x},$$

equation (41) becomes

$$\Delta M_o = (\sigma k/x)N_o e^{-\sigma p/x} \left(\frac{p}{p_o}\right)^{1/\beta c\tau_o \epsilon \lambda x}. \tag{42}$$

Put

$$s \equiv 1/\beta c\epsilon\tau_o \lambda x; \qquad \nu \equiv \sigma p_o/x; \qquad \eta \equiv p/p_o. \tag{43}$$

Then

$$M_o = k\nu N_o \int_o^1 e^{-\nu\eta} \eta^s d\eta.$$

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[15] This neglects loss by ionization and diminution of energy and so of mean life as a result of energy loss along the path. The approximation is justified, for it turns out that the mesotrons with which we shall be chiefly concerned in the present section are slow mesotrons for which $(1 - \beta^2)^{-\frac{1}{2}}$ is not far from unity. Thus the mean life for such mesotrons is not very sensitive to change of energy, and tends to approximate to the minimum value τ_o. The minimum energy of the mesotrons concerned will be about 2×10^7 e.v. We shall not be concerned with water equivalent depths greater than 1 meter of water. For such depths a mesotron would lose about 100 volts per actual centimeter of path, and to lose 2×10^7 volts would require a path of 2×10^5 cm. and a time of 0.7×10^{-5} second. With a mean life of 2.5×10^{-6} second, only a fraction of about e^{-3} of the original mesotrons would survive to end their lives by ionization. In other words, losses determined by mean life considerations are all-important in comparison with losses through ionization.

Integrating successively by parts, and treating s as an integer [16] we find, after a few simple steps,

$$M_o = kN_o ve^{-\nu}\left[\frac{1}{s+1} + \frac{\nu}{(s+1)(s+2)} + \frac{\nu^2}{(s+1)(s+2)(s+3)} + \cdots\right],$$

or, on replacing ν by its value in terms of (43),

$$M_o = \frac{kN_o\sigma p_o}{x} e^{-\sigma p_o/x}\left[\frac{1}{s+1} + \frac{\sigma p_o/x}{(s+1)(s+2)} + \frac{(\sigma p_o/x)^2}{(s+1)(s+2)(s+3)} + \cdots\right].$$

Let us now replace N_o by [17] $(A \sin\theta/\gamma^3)d\theta d\psi d\gamma$. Then the corresponding contribution to the vertical intensity I_V is obtained by multiplying the result by $\varphi(x)$ as defined by (35), and then integrating with respect to ψ from 0 to 2π, with respect to γ from the lower limit γ_m to infinity, and with respect to x from unity to zero. A similar procedure is to be adopted for the horizontal intensity I_H, except that $\varphi(x)$ is replaced by $\varphi(\sqrt{1-x^2})$, and we must multiply by $\cos\psi$ and integrate from $\psi = -\pi/2$ to $\pi/2$. The results are:

$$I_V = 2\pi Ak\sigma p_o \int_o^1 \frac{e^{-\sigma p_o/x}}{x}\,dx \int_{\gamma_m}^\infty \varphi(x) \times\left[\frac{1}{s+1} + \frac{\sigma p_o/x}{(s+1)(s+2)} + \cdots\right]\frac{d\gamma}{\gamma^3}, \quad (44)$$

$$I_H = 2Ak\sigma p_o \int_o^1 \frac{e^{-\sigma p_o/x}}{x}\,dx \int_{\gamma_m}^\infty \varphi(\sqrt{1-x^2}) \times\left[\frac{1}{s+1} + \frac{\sigma p_o/x}{(s+1)(s+2)} + \cdots\right]\frac{d\gamma}{\gamma^3}. \quad (45)$$

[16] s is of the order $1/(3 \times 10^{16} \times 2.5 \times 10^{-6} \times 10^{-6} \times 1) = 100/7.5$. In other words, s is of the order 13. Hence, it is always sufficiently near to an integer to be taken as an integer within the permissible limits of accuracy. When dealing with different values of x, moreover, we can always choose a value such as to make s an exact integer if we choose.

[17] Here A is a constant, θ the zenith angle, ψ the azimuth, and $\gamma = E/m_oc^2$.

As regards γ_m, we first notice that at magnetic latitude of the order of 50 degrees, the entrance momentum for the primaries is largely independent of azimuth and zenith angles, and we shall take it to be the same for all directions. We have $\gamma_m = (1 - \beta_m^2)^{-1/2}$, where β_m is the same for the mesotrons as for the primary particles. β_m is given by (27), with $n = 1$ for singly charged primaries of mass m. The value of (φ) for latitude 50° is $3.31\ m_p$, where m_p is the mass of the proton.[18] Thus for the case of the protons and so for mesotrons born from protons, (27) yields

$$\frac{m_p\beta_m}{(1 - \beta_m^2)^{1/2}} = 3.31\ m_p. \tag{46}$$

This yields a value $\beta_m = 0.96$, and a value $1 - \beta_m^2 = 0.084$.

It is easy to show that, for a single velocity $(=\beta c)$, $\varphi(x) = 1 - \beta^2$ for emission perpendicular to the line of flight of the mesotron, and $(1 + \beta)/(1 - \beta)$ for emission along the line of flight. It is obvious, therefore, from (36) that, for a single beam of mesotrons, the ratio of electron intensity perpendicular to the beam to that parallel to it will be negligible unless β is appreciably different from unity. It is almost obvious, and calculation confirms the expectation, that even when all directions of the primary beam are taken into account, I_H/I_V will be negligible unless β_m is appreciably different from unity. Such a value of β_m as is calculated above for protons would be quite inadequate to account for an appreciable horizontal intensity.

If now we try the particles next in the line of mass to protons, in other words, if we try helium atoms, (46) is replaced by

$$\frac{4\ m_p\beta_m}{(1 - \beta_m^2)^{1/2}} = 3.31\ m_p.$$

This leads to

$$\beta_m = 0.639; \qquad (1 - \beta_m^2)^{-1/2} = 1.30; \qquad \gamma_m = 0.30.$$

[18] The minimum momentum for entry is, of course, independent of the mass of the particle. We have here simply forced cm_p as a factor for convenience of discussion.

We wish now to apply this value in (44) and (45). However, before doing so, it is necessary to know σ. We cannot determine this by a comparison of the observed and calculated values of I_V at different altitudes, because the observed intensity includes mesotrons and electrons born from the proton primaries already provided for in Section III. However, on account of the fact that, for these primaries, β_m is nearly equal to unity, they will make no contribution to the *horizontal* electron intensity. We may thus use (45) to determine the value of α by comparison of the values of I_H at two altitudes. The procedure was to plot against σ the calculated ratio of the values of I_H for water equivalent depths of one meter and two meters, and then find the value of α for which the calculated value of the ratio agreed with the experimental value as given by the second National Geographic U. S. Army Air Corps stratosphere flight (see footnote 19 of Part I). The result was to yield a value $\sigma = 8.0$,[19] for p in atmospheres.

With this value of α, it turns out that $I_H/I_V = 0.22$ for $p = 0.1$ atmosphere and $I_H/I_V = 0.13$ for $p = 0.2$ atmosphere. Taking into account the fact that the horizontally directed telescopes measured twice the true horizontal intensity, the experimental values for the I_H/I_V are respectively 0.21 and 0.13. The closeness of the agreement is, of course, accidental, for in the first case the calculations refer only to the electron intensity, and to the electrons generated by the death of the mesotrons born of the helium atoms. Indeed, the implications of even an approximate agreement of the above kind are that at these high altitudes, the progeny of the helium atoms are predominant in relation to those of the protons. A circumstance which helps in this direction is, of course, the fact that a helium atom gives four times as many mesotrons as a proton, and these mesotrons give a copious supply of electrons in a short distance on account of their low energy and resulting short mean lives. Of course, the low energy of the helium born mesotrons causes the helium contribution to the total radiation to diminish rapidly with descent into the atmosphere in comparison with the proton contribution.

[19] The experimental value of the ratio is 1.8. Incidentally, it may be mentioned that for the purposes of the present calculations, and with the value $\sigma = 8.0$, only the first term in the series in equations (44) and (45) is worth retaining.

V.2
194

Admittedly there is very little justification for assuming an inverse cube for the law controlling the energy spectrum of the incoming helium atoms. However, the results are largely independent of the form of this law, so long as the inverse power is not small, for so long as this is true, the primaries near the lower limit of energy are mainly responsible for the results calculated.

While there is of necessity considerable uncertainty in the details as regards the matters here under discussion, the broad fact emerges that, in order to obtain horizontal intensities comparable with those observed, the only obvious source of explanation is to be found in the electrons ejected on the death of mesotrons, and to secure from these a sufficient horizontal intensity, the mesotrons must be of energy too low to permit of their having been born from protons which could penetrate the magnetic field. One needs a heavier particle, such as a helium atom, which, in virtue of its greater mass, can penetrate the field with smaller velocity than would be possible for protons, and which can, therefore, hand on to its mesotron progeny that small velocity necessary to secure the asymmetry.

One might be inclined to suppose that the magnetic field could play a vital role in bending the paths of electrons and resulting in an appreciable horizontal intensity. However, quite apart from the fact that calculation does not substantiate the expectations along this line—partly on account of the fact that the lines of magnetic force are not far from vertical in the regions concerned—the experimental data show no appreciable variation of horizontal intensity with azimuth such as would be expected if the magnetic field played a prominent role in this connection.

In conclusion, the writer wishes to express his appreciation of the discriminating services of Mr. Paul Weisz, who has performed most of the numerical computations in this paper.

One Hundred Degrees Below Zero.—JAMES G. MACORMACK. (*Refrigerating Engineering*, Vol. 45, No. 4.) Climbing to high altitudes—35,000 feet and up—is the challenge to our fighting and transport planes and it must be met. It is logical to conclude that the Air Force which gains control of high altitudes will gain supremacy in the air. Just as wind tunnels are used for testing correctness of structural and aerodynamic design, so altitude testing chambers are used for testing actual performance of planes, men, instruments and equipment under conditions to be encountered. Altitude testing chambers are designed to reproduce on the ground the conditions of rapidly changing temperature and air pressure encountered by planes and their occupants during climbs to high altitude. The problem of furnishing the low temperatures for these test chambers was one for the refrigeration industry. Not an easy one it seemed when the specification of dropping from + 70 deg. F. to − 100 deg. F. in approximately 12 minutes was introduced. The previous accepted lowest temperature of − 67 deg. F. in the stratosphere was found incorrect when temperature readings in the range of − 95 to − 105 deg. were actually taken from planes. Two types of test chamber have been designed and constructed. The first an altitude test chamber where low temperature and vacuum conditions are produced simultaneously. This necessitates a heavy steel vacuum shell to withstand outside air pressure (14.6 lb.) when pressure inside the chamber is reduced to 2 lb. or less, the pressure encountered 9 miles above the earth. The second type is a static cold chamber where only low temperature is required and this of course needs no pressure requirements. This type of chamber is used in tests of materials that may be affected by low temperature only and not by low pressure. Mechanical refrigeration is being used in the majority of both types of installations with Freon-12 as the usual refrigerant.

R. H. O.

THE SONGS OF INSECTS.*

BY

GEORGE WASHINGTON PIERCE.

This highly honored Award, the Franklin Medal, has roused my heartfelt gratitude to the President, Directors and committees of this bountiful and friendly organization. I am greatly cheered and encouraged by the generous words used in the President's and Sponsor's citation, and I can wish for nothing higher than that my career had to a slight extent merited so gracious an encomium. I am especially happy to have my name listed among the great and distinguished scholars variously honored here tonight and also with those previously placed on your Roll of Medalists. To see how unworthy I am to be included with them, you only have to recall the well-known fact that a physicist knows no engineering and an engineer knows no physics, and that the President of Harvard University characterized me by making me professor of both subjects. Your wise patron Benjamin Franklin said, "If a man empties his purse into his head, he has that which no man can take away from him." The Franklin Institute here tonight has been emptying its purse into other men's heads, which seemed also a good practice.

I have greatly enjoyed this meeting. Some doubt might exist as to whether Philadelphia is the Cradle of Liberty, but it is certainly the trundle bed and hospitality center to which all liberty-loving persons come as soon as they can walk.

The notice of this Award reached me at our home in Florida. While I was spending a sabbatical leave in Florida, several years ago, my late good friend General Carty asked me how I could get away from Harvard for such a lengthy absence from my duties. I told him, "I have got so far ahead of my science I had to come away for a year to let it catch up." I see now that such a statement was mere persiflage and idle boasting. My long absence from books and knowledge, coupled with a failing memory, is poor preparation for this address, and I have with me not a single recorded observation or experimental note as a basis for this address on

* Read at the Medal Day Meeting, Wednesday, April 21, 1943.

the "Songs of Insects." Pardon me, therefore, if I omit most of the numerical and historical facts, and if my delivery in the absence of notes and motion pictures is confined largely to pantomime.

My work on insect noises and tones has been a pastime of many summers. It was begun in complete ignorance of entomology. Many of you have spent hours of sleeplessness unwillingly listening to a chirping cricket, but few of you have ever observed him in the act of chirping. The cricket has no voice and makes his noise strictly by stridulation; that is, by lifting his wing covers and scraping the edge of one wing cover on a row of teeth on the other wing cover. This means he rubs the edge of one wing cover (the scraper) over the teeth of the other wing cover (the file). He has scrapers on both wings and files on both wings, but a *given species* of insect in stridulation always places the file of (say) the *right* wing in contact with the scraper of (say) the left wing. He might be called right-winged; while another different species might be left-winged. Both types occur.

Of course, there are other methods of noise-making. For example, many grasshoppers stridulate by rasping the thigh members of both hind legs over the edge of the wing covers. The thighbone has a file on it and those files rub the edge of the wings.

There are other methods of noise-making which I will mention later, if there is time. In describing the songs I will include only the cases where the song is made by some particular instrumentality adapted to the purpose of song-making. That is, we won't pay attention to a sound of an insect hitting the window pane—you can also hear some of them chewing, for example—but all those noises I am classing as mere incidental noises. The songs, however, are definite and nearly constant in each species. That is, they differ from species to species. They may be accurately described and accurately measured.

Some of these songs—excuse me for calling them "songs"; they are not songs—for the insects have no voice but their tones may still be really musical. Now some of these songs have a fundamental pitch entirely above the limit of audition by the human ear; that is, the normal human ear can hear up

to about 18,000 vibrations per second. With older people the upper limit may be only 10,000 or less, but even with the normal unaided ear alone, the main notes of some of these insects can not be heard and have never been heard before these experiments began, as far as I know. However, I make this qualification, if a good listener places his ear near such an insect when he is emitting a cry, he will hear a feeble scratch, similar to what you hear when you scratch your finger across a piece of clothing—but that scratch is not the insect's note at all; it is merely incidental.

Now with a detecting apparatus for receiving supersonic vibrations of sound in the air above the limit of audibility, such an instrument having in it the means for transposing pitch, you transpose from this high inaudible frequency down to the audible frequency. That's the trick of my experiments. If the pitch you have is 30,000 vibrations a second, you can make it audible by subtracting 25,000 cycles per second. You understand if you have two tuning forks making notes nearly alike, or two piano keys, you can hear beats between them. You might get one beat a second or five beats a second, if the difference between the two notes is one or five vibrations a second, but you can't do that if you have two sounds above audibility. That is, if you have the 20,000 and the 19,000 cycle notes, and you play them both at once, you can't hear them, nor can you hear the difference-note of 1000 cycles per second. The reason you hear beats when the two vibrations, both audible, are going on at once, is as follows: if the vibrations get out of phase there is no sound at all, and when the sounds come back in phase you get sound, and it is that fact, that the sound increases and decreases, that makes the beat note of audible sounds audible. But, with higher frequency above audibility the beat note or difference-tone is not audible. But by using the electronic tubes the difference-tone between superaudible sounds becomes audible through a loud-speaker.

Electronic vacuum tube devices may be arranged so as to produce electric currents and mix the electric currents in a tube of certain characteristics; then you may get out a tone which is the sum or difference of the frequencies of two sounds—so the machine I am going to describe briefly has a

method of rendering an inaudible sound audible by converting to electric current and applying to the system, an oscillation by an internal oscillator of a frequency different from the first one, then separating out the difference-note as an audible sound. That is, you can transpose notes all over the scale mechanically by using an electronic device. You will be glad I haven't time to go into a description of this apparatus. I can't leave it alone entirely, though.

It has a small horn, about a foot long and six or eight inches in diameter, which is parabolic, and has a detecting instrument at the focus of the horn. Now the detecting instrument must be a kind of microphone which responds to vibrations of the pitch of the sound you wish to detect. An ordinary form of microphone can be adapted to this purpose, but it is much simpler to use the piezo-electric crystal. For that purpose a piece of Rochelle Salt has been employed. This piece of Rochelle Salt, which should be made of the right length and size, is characterized by the property that if the sound falls on the end of it or any pressure is applied to the end of the salt, it develops an electric voltage on two metal plates on the sides so as to convert mechanical vibrations into electric vibrations.

So in this apparatus the detecting apparatus, this device of Rochell Salt, was employed for converting sound energy into electrical energy. This electrical energy in this machine goes through a tuned circuit adapted to determine the frequency, which is read on a dial of the tuning instrument. Now, after all that, these currents are amplified and finally converted into sound coming out of the loudspeaker, in the audible range. That is, the inaudible insect sound becomes audible sound.

This device has also attached to it a photographic recording instrument which writes on paper the code or message of the insect's song. The result is that with such an instrument placed on a raised platform, as I had, in an insect-infested terrain, the noise-maker may be located by the direction of the pointing of the horn. The frequency is determined by the adjustment of the tuning dial. Turn the tuning dial and you get a lot of sound, so you regulate the tuner and note what the pitch is or what the number of vibrations is,

V.2
194

and you know the fundamental of his sound. Now with this instrument for locating the direction of the horn, the observer or a stealthy assistant may go out in the field and climb an appropriate tree and collect the specimen, which may be examined, classified and made to pose for his photograph. We have motion pictures taken of these insects while stridulating, making their noise. This motion-picture taking required a good deal of practice, because sometimes we had to use so much light, especially in taking colored motion-pictures that the insect got hot, so we had to put the insect on a wet pad to keep him cool, while we took these motion pictures. You can see their wings flapping and I have studied some fifty species in captivity and in the field in this manner. Some excellent motion-picture work has been done by Dr. Lutz and many of the results of my work have been previously obtained by him.

I will tell only a few things about a few of these insects. The ordinary cricket, if you want to talk about that, makes a chirp by scratching one wing across the other and the pitch is determined by the number of teeth on his wing and on the rapidity of motion of the wing, so after you get the pitch you can find out just how many teeth are in contact during this stridulation.

Now a cricket goes "Chirp, Chirp, Chirp," as you know. Each chirp is found to be about three notes very close together. So if as with my apparatus we rectify the high-frequency currents and make an oscillogram of them plotted against time, we have the duration and the character of each pulse. Then by reading on the tuning dial the frequency or the pitch of the sound, we have an accurate record of the sound made. The ordinary house cricket makes about 4900 vibrations per second. That is a long way up in the musical scale, somewhat higher than the highest note of a piano.

There is a cricket a little smaller. He is called nemobius—he is about half an inch long and so plentiful in some localities that the landscape is dotted with one to every square foot over a whole field. I first heard one of the nemobius insects by putting a listening device out of the window at night and finally pointed the pointer horn around in different directions until I found that the sound got loudest

when the instrument pointed in a particular direction toward a spot 200 feet away. When I walked down there and listened intently I could hear him down there and at the same time I could hear the loudspeaker giving out its note 200 feet back at the house.

There are three or four different sub-species of these nemobius crickets which are practically exactly alike to microscopic and entomological investigations, but the different subspecies make distinctly different sounds. By collecting several hundred of them we separated these several hundred according to their songs, and carefully examined them microscopically. The only difference we could find was in the number of teeth of the files on the wing, a very slight difference, but sufficient to characterize them into three or four groups (subspecies). An observer with a proper sound measuring apparatus may sit on his piazza and identify and classify sound-making insects by observing the pitch and code of his singing.

I might say the crickets and many insects have what seem to be ears, or an auditory device, on the fore-arm joint of their front legs. For some species of insects, the male makes a strong sound and when he makes his sound you will hear the female respond by making a very slight click, click of very high frequency.

I once had a very interesting experience with a northern katydid. We were attempting to get a moving picture photograph of the katydid while singing, but we could not do it because he wouldn't sing in the daytime or in the light, although he would sing beautifully at night. So we first made a phonograph record of the singing katydid at night, and played it to him in the daytime, and he started up. We found that was not necessary, because after some experience we found that we could make almost any noise we wanted to and after a while he would start up—we educated him. He had been making only two pulses, going "Katydid, Katydid" and we shouted at him and finally got him so he would make a call of four pulses. Finally we got him up to five pulses, sounding like "whank, whank, whank, whank, whank," and then he lost count. So by kindness we find that we can educate even a katydid.

THE USE OF MODELS AS A PRACTICAL AID IN THE DESIGN OF SUSPENSION BRIDGES.

BY

FRANCIS P. WITMER.

For about ten years studies have been in progress at the University of Pennsylvania involving the determination of stresses in the stiffening trusses of suspension bridges by the aid of a mechanical model. The purpose was not merely to

View of Structural Research Laboratory, University of Pennsylvania, showing suspension bridge model.

verify stresses computed analytically, but to determine whether stresses obtained with a model might not be considered sufficiently accurate to be employed for actual design, with a consequent saving in the time, labor and cost of such design.

The studies first made were embodied in a thesis for the degree of Sc.D. in Engineering by Francis Lee Castleman, Jr. This model comprised the main central span only of a suspension bridge, but with provision by which there could be included the effects upon this span of changes in length of side span cables, resulting from stress and temperature variation. The model did not produce moments for side span trusses themselves, but it could be adapted for such side span study, without great difficulty.

The cable used by Mr. Castleman was a piano wire of assumed convenient diameter, whose elastic properties, in comparison with those of the cable in the bridge under consideration (the Manhattan Bridge in New York) were made the basis for the computation of the proper loading on the model, in order to preserve similitude with the loading on the bridge.

From the relative loads on bridge and model as thus determined the elastic properties of the model stiffening girder were next computed to be equivalent to those of the bridge trusses. The model girder was of wood of uniform quality, carefully finished to calculated dimensions, and whose modulus of elasticity was determined by a bending test. It was suspended from the cable by wire hangers with provision for convenient adjustment in order to obtain their correct lengths for any desired cable sag.

V.2
194

Thus, the entire design of the model including its cable and girder and the dead and live loading to be placed upon it, were in practical harmony with the corresponding quantities in the Manhattan Bridge. This similarity was not carried to the design of the suspenders, except as regards their correct length, as their precise sectional area did not materially affect the resulting moments in the girder.

Positive moments were obtained by the model at the quarter point and at the center of the span, and also the shear in the end panel, these taking into account the effect of live and dead loads alone, and also the effect of these loads in combination with the effect of temperature change.

The results were quite satisfactory, being generally within 2 per cent. of those computed by the deflection theory, with a maximum difference of 6 per cent. in the case of the end shear inclusive of temperature.

The connections of suspenders to the wire cable were somewhat difficult, and furthermore the piano wire had a high resistance to bending and thus violated a theoretical assumption of the deflection method of analysis. It was therefore decided to substitute a cable made from beaded chain such as is frequently used for operating automatic furnace dampers and for other similar purposes. This would afford a ready method of connecting suspenders to the cable wherever desired by means of a wire hook between beads, and the cable would also be practically free from bending resistance, and thus more closely in accord with the deflection theory.

This cable of course was frankly lacking in elastic similarity with that of the bridge. The stiffening girder, therefore, could be made of any convenient dimensions, and the dead and live loads for the model computed by a consideration of the dimensions of bridge and model, the loading on the bridge and the elastic properties of the bridge trusses and of the model girder.

The diameter of a hypothetical wire cable was next computed such that if it were used similarity would be maintained with the bridge cable, by a proper consideration of the relative loads just determined. The quantity (AE) for this wire cable was then used in comparison with a similar quantity for the beaded cable, obtained by a tensile test of distortion under a known load, and from this comparison a correction was derived for use in conjunction with the beaded cable, complete similarity with the bridge thus being restored.

Results obtained with the model will first be given, and after that, a brief description of the model itself with reference to its more important details, many of which required considerable experimental trial before their most advantageous forms were finally determined. In the entire work the writer was ably assisted by Harry H. Bonner, M.S., of the faculty of the Frankford High School in Philadelphia.

The first comparison made was with the same points of the Manhattan Bridge as were studied by Mr. Castleman, but only the cases without temperature effect were considered.

Table I gives the comparative values of positive girder moment obtained by the model and by the deflection theory, for the Manhattan Bridge.

TABLE I.

Manhattan Bridge Stiffening Truss.

Panel Point.	Maximum Moment (without Temperature). Deflection Method.	Model.
1 (for end shear)	+48,368,000 ft.-lbs.	+50,700,000 ft.-lbs.
4 (quarter point)	+99,600,000	+97,800,000
8 (center point)	+62,149,000	+56,500,000

Next a study was made of the Bear Mt. Bridge, with temperature again omitted. The model was re-assembled to simulate this bridge as to dimensions and loading, using the same stiffening girder and beaded cable as for the preceding case.

Fig. 1

Bear Mountain Bridge Stiffening Truss.
Positive Moments—(without temperature)
(a) By Elastic Method
(b) By Model
(c) By Deflection Method

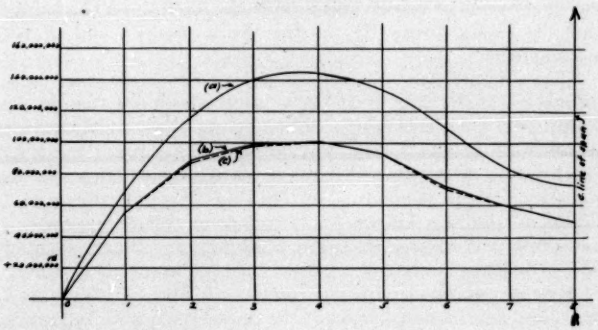


Figure 1 shows:

(*a*) Curve of positive moments of Bear Mt. Bridge, computed by the elastic method which was used in the original design.

(*b*)(*c*) Curves of positive moments of Bear Mt. Bridge computed by the deflection method, compared with those obtained by the model.

Figure 2 shows curves of negative moments of Bear Mt. Bridge and model, similarly obtained.

Finally a study was made of the Philadelphia-Camden Bridge, with temperature effect included. Here the comparison was with stresses given on the contract drawings for the bridge. These were not verified by further analytical study, but were assumed to be correct. Both cable and stiffening girder were the same as for the Bear Mt. and Manhattan studies, but new model loads were of course computed.

Figure 3 shows:

(*a*)(*b*) Curves of positive moments derived from stresses given for the Philadelphia Bridge, compared with those found from the model.

(*c*)(*d*) Curves of negative moments of Philadelphia Bridge and model, similarly obtained.

In making the studies of the three bridges considered the cable and stiffening girder were kept unchanged, but suspender lengths were varied to produce the proper parabolic ordinates of the cable for each bridge, under dead load alone. Under this condition the girder was considered to be free of bending stresses. The correction for the beaded cable was necessarily calculated for each bridge.

The comparisons for Manhattan and Bear Mt. Bridges were made with moments computed by the deflection method, at the identical points corresponding with panel points of the model, and the loading conditions were also identical, those for Bear Mt. being obtained by a consideration of the influence curves produced in the original elastic analysis. These curves for moment are shown in Fig. 4.

For the Philadelphia Bridge no stresses were given upon the contract stress sheet at points corresponding precisely with the panel points of the model which, for all cases, was constructed with 16 panels, and the moments on Fig. 3 were therefore interpolated as could best be done from the figures upon the stress sheet. Furthermore, no information was available showing the loading conditions for the Philadelphia Bridge analysis, i.e., the panels covered by live load, and the conditions assumed for the model were therefore, those indicated by the Bear Mt. influence curves in Fig. 4.

The following advantages possessed by the model are worthy of special emphasis.

Fig. 2

Bear Mountain Bridge Stiffening Truss

Negative Moments—(without temperature)

(a) By Elastic Method

(b) By Model

(c) By Deflection Method

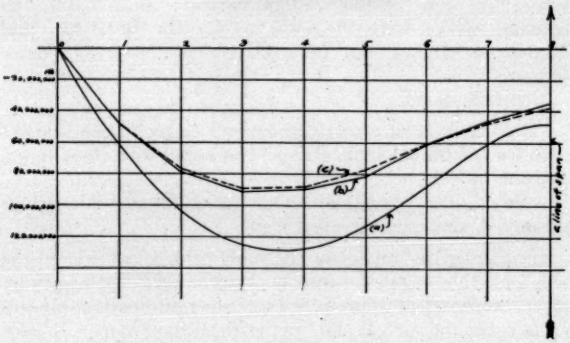


Fig. 3

Philadelphia Bridge Stiffening Truss

Moments—including temperature where this controls section—(computed)

(a) Positive Moments by Stress Sheet

(b) " " " Model

(c) Negative " " Stress Sheet

(d) " " " Model

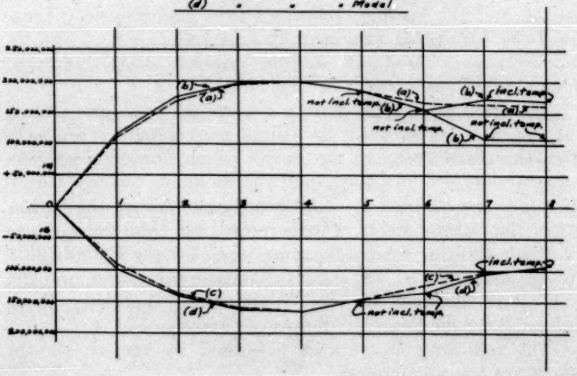
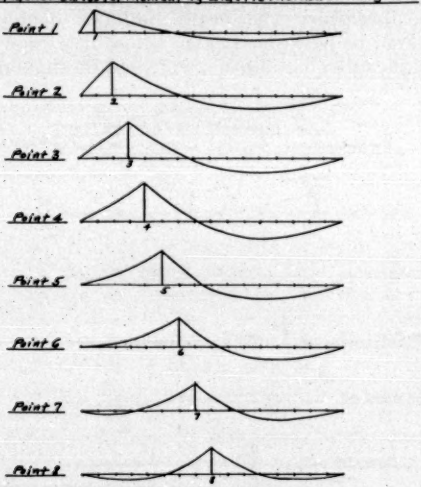


Fig. 4.
Bear Mountain Bridge Stiffening Truss
Influence Curves for Moment by Elastic Method of Analysis



(1) The same cable and stiffening girder, and the same suspenders may be used for any case which it is desired to study, the suspender lengths being adjusted as required.

(2) The dead panel loads (in addition to the weight of cable, suspenders, girder, and all connections), may be readily computed by a consideration of bridge loading and the girder constants of bridge and model.

(3) The live panel loads may be computed by a similar comparison.

(4) The re-assembly of model for a new study may be made very quickly, the proper elevations of cable at various panel points under dead load being obtained by the use of an engineer's level. Model dead and live panel loads must also be computed, together with the corrections to be applied because of the use of the beaded cable.

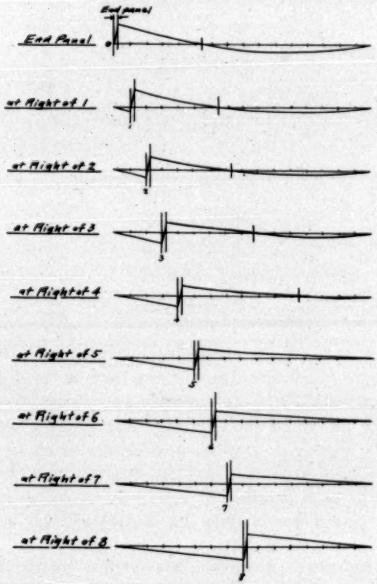
(5) The position of live load for maximum moment at any point may be found quickly by a few trials with the model.

By the deflection method this position must be assumed by an exercise of judgment, and if a second assumption is considered necessary the entire arduous computation of moment must be performed again before it is known whether the one or the other assumption will yield the maximum value.

Fig. 5

Bear Mountain Bridge Stiffening Truss

Influence Curves for Shear by Elastic Method of Analysis



(6) An entire series of moments for one-half truss can be found by the model in a few hours, and can be checked and re-checked as often as desired. Generally the results by repetition are in such close agreement as to be actually startling. Any considerable difference may usually be diagnosed as due to some error in observation or some lack of customary

smoothness in the working of the model, the cause of which can generally be quickly found and remedied.

(7) Shear in any panel may be found by a comparison of the moments at the two ends of the panel. Thus, in the panel between points (3) and (4), the following relation will suffice:

$$M_4 = M_3 + Vp - \frac{wp^2}{2}$$

or

$$V = \frac{M_4 - M_3}{p} + \frac{wp}{2},$$

where M_3 and M_4 are the moments at points (3) and (4) respectively, V is the shear sought, w is the live load per linear unit and p is the panel length. This assumes the panel (3)–(4) fully loaded. Loading condition for maximum shear can be found by a few trial computations with the model. The influence curves of shear, resulting from the elastic analysis for Bear Mt. Bridge, and shown in Fig. 5, will be found useful in guiding the judgment for such trials.

If the reader is sufficiently impressed by the results given in the foregoing, he may be impelled to consider the following more detailed description of the model itself.

DESCRIPTION OF MODEL.

Figure 8 shows in a single sketch the principal parts of the model as hereafter described.

As before stated the model covers only the main center span of a suspension bridge. The effect of side span action upon the center span may be readily included in the operation of the model, by a simple adjustment. No towers are used, as, for all practical purposes, these produce no effect upon the center span, except as regards the horizontal movement at the top, from stress and temperature action upon the side span cables, and the adjustment just referred to takes care of this. The span therefore is hung from a pair of horizontal $2\frac{1}{2}'' \times 2\frac{1}{2}'' \times 3/16''$ steel angles about 16 ft. long, bolted together back to back, with 1″ thick washers between the upstanding vertical legs. These angles rest upon pedestals at their ends. The floor and pedestals must be quite free of

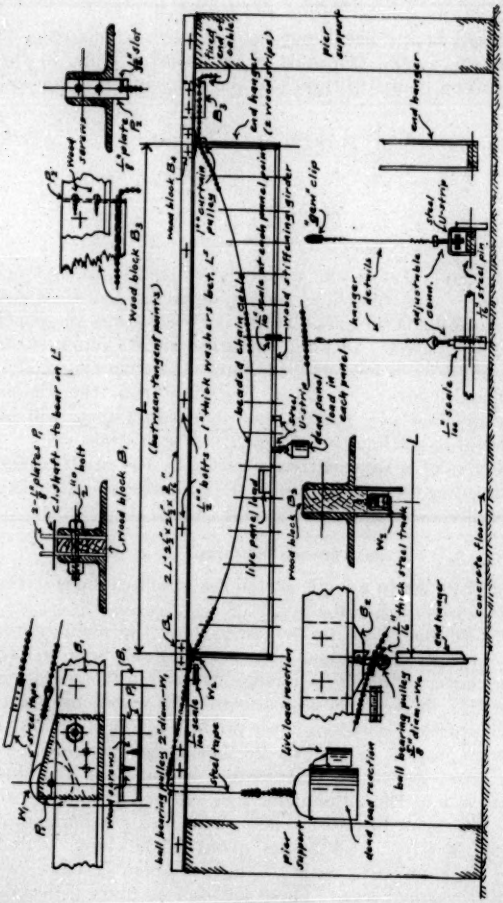


Fig. 8
Principal Details of Model

V.21
194

vibration from extraneous causes, for satisfactory behavior of the model.

The cable is permanently connected to the under side of the angles at one end, and at the other passes over a highly sensitive ball-bearing pulley about 2″ in diameter, the vertical end of the cable, after leaving this pulley, carrying a bucket in which are placed weights to produce a reaction for balancing properly the loads applied on the span.

In order to define precisely the effective length of the model span, this is regarded as the distance between tangent points at which the cable passes over a small pulley firmly connected under the angles, at each end. At the fixed end this is an ordinary brass curtain pulley about 1″ in diameter, but at the free end it is a highly sensitive ball-bearing pulley 5/8″ in diameter. The latter must receive a small positive reaction from the cable at all times. In other words, the cable must never be permitted to rise free of the pulley by reason of an upward deflection of the cable at this end, caused by live load on the span at the further, fixed end. This is necessary in order to define precisely the length of the span. To reduce resistance, this reaction should be as small as possible. The cable, therefore, has a slight break from a straight line at the point of tangency, and approaches the main reaction pulley at a small angle with the horizontal. This condition makes it convenient for the cable to pass between the angles at this end, the main pulley being connected on top of the latter.

In order to ensure as free an action as possible, the beaded cable is interrupted a few inches before reaching the larger pulley at the free end, and is replaced by a length of steel tape about 5/16″ wide which takes the bearing upon the pulley, instead of permitting the beads to bear directly. For a similar reason, a narrow strip of steel about 1/16″ thick is used as a track between the beaded cable and the smaller ball-bearing pulley, being bent to a gentle curve to permit smooth movement of the cable. This strip should be shifted to a new point of bearing frequently for best results.

Between the two ball-bearing pulleys there is attached to the cable a short length of 1/100″ scale, placed in a horizontal position, which, when observed through an engineer's transit,

located as close to the model as will permit proper focusing, shows, with an observed accuracy of 1/10 of a division, or 1/1000″, the amount of horizontal movement of the cable at this point, due to any variation of applied load or bucket load.

The complete bucket load, including the weight of the vertical portion of the cable below the main pulley, equals the stress in the span cable immediately adjacent to the point of tangency on the small pulley. This stress is readily converted into a value of H (the horizontal component of the cable stress) by a simple trigonometric computation. Thus, by proper consideration of the position of this scale and adjustment of bucket load there is obtained the value of H for any condition of loading on the span, including, by a properly calculated movement of the scale, the effect of side span cable stretch and of temperature, together with the correction required for the beaded cable. The sensitiveness of the pulley action enables an extremely accurate determination to be made—within the limit of about 10 grams in a total bucket weight of perhaps 20000 grams, which means an error in the value of H of between 5/100 and 1/10 of one per·cent.

From the cable, at panel points corresponding to 16 panels in the model span, the suspenders are hung. These are connected to the cable by means of thin wire hooks. The connection is made between the two beads which are most nearly above the particular panel point of the stiffening girder which is under consideration. A slight error horizontally in the location of this point of connection to the cable causes no serious error in the moments determined by the model.

The suspenders are of thin wire connected at the bottom to the stiffening girder by a simple device with a bolt and nut which admits of a ready and precise adjustment for length of hanger.

Additional dead panel loads to agree with the computations are hung under the girder at the center of each panel.

At each panel point there is attached to the girder a short vertical 1/100″ scale so arranged that it may be rotated slightly about its supporting pin to enable it to be made truly vertical under any position of the elastic curve of the girder when subjected to live load or temperature. This is necessary for accurate reading of girder deflections. These

are made by means of an engineer's level placed as close to the model as proper focusing will permit, and here again, readings may be made to 1/1000", as the magnified size of the 1/100" divisions permits a satisfactory estimate to 1/10 of a division.

When the full dead load is on the span, the girder should be horizontal, if it was originally straight. For this condition, the value H_d of the horizontal component of cable dead load stress is found by the familiar formula

$$H_d = \frac{wl^2}{8h}, \qquad (1)$$

where w = dead load per unit of span length,
l = span length,
h = center sag of cable, with reference to a horizontal line between tangent points.

The necessary weight is placed in the bucket to produce this computed value of H_d and under these conditions, with full dead load on the span, the cable panel points must be carefully adjusted by modifying the hanger lengths until, with the girder level, the cable will have the correct elevation at each panel point, and will thus hang in a true parabola. This is done by observation with the level, hanging a vertical 1/100" scale of sufficient length at each end point of tangency and at each hanger cable connection in turn, and adjusting hanger lengths until the proper computed cable elevation, with reference to the horizontal line between tangent points as a datum, has been obtained. This operation takes considerable time and patience. The ends of the girder are connected to vertical wooden hangers, conveniently attached by pins at the top. The bottom pin connections of these hangers to the ends of the girder determine the elevation of the latter. This elevation will be the same at the center point of the span, and, by making the length of the center hanger as nearly correct as possible, the elevation of the center point of the cable may be computed, to make it agree with the correct sag of cable at center with reference to the datum line above mentioned. Having located this center point properly, other points of the cable must then be adjusted in

turn. It is more accurate to place the correctly computed load in the bucket and thus ensure the proper cable center ordinate, rather than to attempt to set the cable precisely to this ordinate and then obtain the bucket load and value of H_d by experiment.

Having properly adjusted the model for dead load as above described everything is in readiness for its use in determining moments from live load and temperature. For this purpose live panel loads are to be placed on the girder at the middle of such panels as it is desired to load for the moment sought.

In order to find H from the end cable stress, which equals the load W_b in the bucket, it is necessary to know the sides of the triangle formed by the length along the cable from the tangent point to the first panel point (assuming the curve to be equal to the chord), the vertical deflection of this panel point below the tangent point, and the horizontal projection of the cable chord. Fig. 6 shows these parts

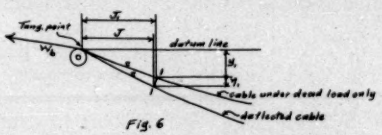


Fig. 6

J_1, y_1 and c are the sides of the triangle in question under dead load, y_1 being the parabolic ordinate for the first panel point (1).

Under live load or temperature action, (1) deflects through a vertical distance η_1 and the triangle, with the total bucket load W_b, becomes one with sides J, $(y_1 + \eta_1)$ and c (neglecting a negligible stretch of the cable in the end panel).

Then $c^2 = J_1^2 + y_1^2$.

$$J = \sqrt{c^2 - (y_1 + \eta_1)^2}$$

and the value of H for the bucket weight W_b from dead and live loads is

$$H = W_b \frac{J}{c}. \qquad (2)$$

Thus, by finding W_b and η_1 the value of H readily follows:

W_b is found by observations on the horizontal scale by the transit; η_1 is found by observations on the vertical scale at (1) by the level.

If we desire the moment M_4 at say point (4), we may write:

$$M_4 = M_1 - H(y_4 + \eta_4), \tag{3}$$

where y_4 = parabolic ordinate at (4),
η_4 = live load deflection at (4),
M_1 = bending moment at (4) due to dead load and live load on the span, assumed acting as a simple span,
H = total horizontal component of cable stress due to dead load and live load and also temperature if desired.

The procedure is as follows:

1st. Find, by level, the dead load elevations of (1) and (4) by reading the respective vertical scales at these points.

2nd. Set transit so that vertical cross-hair is at some convenient point on the horizontal scale.

3rd. Place live load on the span at the proper panels to produce the maximum moment at the panel point to be studied. This will move the horizontal scale through some distance. If now, side span cable stretch and temperature were neglected and if the cable were dynamically similar to the bridge cable, it would only be necessary to add weight in the bucket until this scale moved back to the position first noted by the transit. The value of H found by equation (2) would then be the H for the dead load plus the live load on the span, W_b being the total weight in the bucket, namely, that originally there due to dead load, plus the additional weight due to live load as measured by the horizontal movement of the scale.

If however, the end span effects of stretch and temperature, and also the correction for the beaded cable are to be included the transit must be set so that the vertical cross-hair is at a point nearer the span than the original dead load setting by an amount K which must be determined from equation (4) following. This is equivalent to running the center span cable into the span, over the pulley through this distance K, and the added weight in the bucket will then produce a value

of H which will be inclusive of the effects just mentioned—in other words the true H for the dead load, live load and temperature, and, in addition, the effect of side span cable stretch and correction for the beaded cable. This will, therefore lead to the correct total moment sought. The backstay stretch and a higher temperature will tend to *reduce* the value of H as compared with its value if these effects were neglected.

4th. The weight in the bucket will now be the original dead load reaction W_d plus the added weight for live load and temperature, W_l. *Before removing* W_l from the bucket for the purpose of weighing same, the elevations of points (1) and (4) under dead load plus live load must be taken by the level. The difference between these elevations and those for dead load alone, taken under the first operation, will be the values of η_1 and η_4 respectively.

5th. Weigh the load W_l and add it to the dead load W_d originally in the bucket to obtain the total value of W_d and compute H by equation (2) and then the moment desired M_4 by equation (3).

W_b will usually be measured in grams and η_1 and η_4 in inches. As M_1 is conveniently expressed in foot-pounds of moment in the bridge it will of course be necessary to transform the units so that the term $H(y_4 + \eta_4)$ is in foot pounds. A transforming factor for H and another for η can be readily derived so that readings from the model may be made to result in a value of M_4 for the bridge.

The correction K above mentioned will now be considered. This is found by equation (4) below.

$$K = W_l B \pm T - W_l \left(\frac{l_1}{(AE)_B} - \frac{l_1}{(AE)_W} \right). \qquad (4)$$

where W_l = added weight in bucket for live load and temperature,

B = length of the center span cable to be run over the pulley on account of the cable stretch in the side span, which would be produced by a live load of one gram in the bucket,

T = similar length due to the temperature change in length of the side span cable. The sign is (+) for a higher and (−) for a lower temperature.

l_1 = length of cable from its fixed end to the horizontal scale,

$(AE)_B$ = elastic constant for the beaded cable,

$(AE)_W$ = elastic constant for the hypothetical wire cable which would properly simulate the cable in the bridge, A being its area and E its modulus of elasticity, assumed the same as that of the bridge cable.

If δ_S = stretch of 2 backstay cables from a weight of one gram in the bucket and α = angle of backstay with the horizontal, $B = \delta_S \sec. \alpha$, i.e., it is the horizontal movement of the top of the tower due to this stretch from a bucket weight of one gram.

This relation is shown in Figure 7.

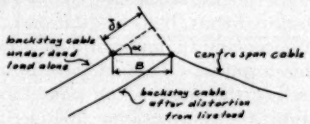


Fig. 7

Similarly if δ_T = change in length of 2 backstays from temperature, $T = \delta_T \sec. \alpha$.

The quantity $(AE)_B$ may be found by the equation

$$(AE)_B = \frac{Pl_1}{S}, \qquad (5)$$

where S is the stretch in a length l_1 of the beaded cable produced by a load P. It may be readily obtained before assembling the stiffening girder and hangers by drawing the cable out into a straight line, and observing with the transit the movement S of the horizontal scale for a load P placed in the bucket.

It will be noted that the quantities H and T are for *2 backstays*, as the entire correction for both towers must be concentrated at one end of the model. If the side span cables are of different lengths, this must be taken into account.

It is convenient to use grams for all loads and bucket weights. The sensitiveness of the model is such that a

movement of the horizontal scale can be detected for an added weight of four or five grams in the bucket and, as before stated, a satisfactory measurement can be made of W_b within an accuracy of about 10 grams, in a total bucket load of 20000 grams or more.

In the model used for the studies described it was found that a slight allowance for frictional resistance should be made in the term $H(y_4 + \eta_4)$ in equation (3). As measurements were always made by adding loads in the bucket, the movement was always from the span, and therefore the observed value of W_b required a slight increase. It was found that this allowance was about 0.2 per cent. so that the term in question was always multiplied by 1.002 before combining with the term M_1. The amount of allowance should be ascertained by experiment, in the case of any particular model.

It should be noted that, before equation (4) can be solved for K it will be necessary to determine an approximate value W_l for use in this formula. This may be done with sufficient accuracy by finding the load which would balance the live load, disregarding the corrections included in K. Great exactness in the value of W_l is not necessary for use in (4).

The mathematical procedure in the design of the model will now be considered.

Constants for the bridge will be assumed as follows:

L = span length between tangent points in inches,
W = an assumed total load in pounds on span, uniformly distributed,
E = modulus of elasticity of stiffening truss, in pounds per sq. inch,
I = moment of inertia of stiffening truss in inches^4,
Δ = deflection at center of span in inches, due to W.

Constants for the model will be assumed as the same letters, in lower case.

All linear dimensions of bridge and model will be in the proportion L/l.

Then

$$\Delta = \frac{5WL^3}{384EI} = \frac{L}{l}\left(\frac{5wl^3}{384ei}\right)$$

V.23
1943

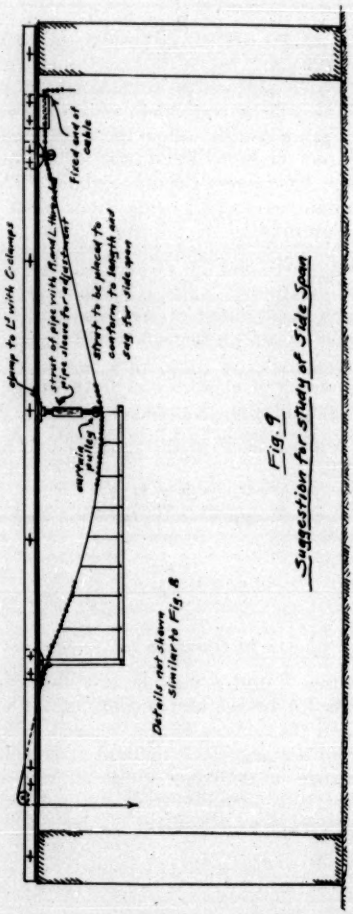


Fig. 9
Suggestion for Study of Side Span

whence

$$w = \frac{WL^2ei}{l^2EI}. \qquad (6)$$

Equation (6) thus gives the total weight in pounds upon the model which will be equivalent to a total load W upon the bridge. Thence readily follow the additional panel loads for dead load and the panel live loads, for use on the model. It is convenient to convert these to grams. The weight of cable, girder, hangers and all connections must be properly considered in determining the additional dead panel load.

The hypothetical wire cable for use in equation (4) may be found as follows:

A = area in sq. inches of bridge cable,
a = area in sq. inches of model hypothetical wire cable,
E = modulus of elasticity of both bridge and model wire cables, in pounds per sq. inch.

Then, for a stress of P in the bridge cable and p in the model cable,

$$\frac{PL}{AE} = \frac{L}{l}\left(\frac{pl}{aE}\right),$$

whence

$$a = \frac{pA}{P} \qquad (7)$$

and

$$(AE)_w \text{ in equation (4)} = aE.$$

The quantities P and p may be any two corresponding loads in pounds for bridge and model, as, for example the total load W and the derived load w as used in equation (6).

Figure 9 shows a suggested method of providing for the study of moments in stiffening girder of side span. This method has not been actually tried out in practice, but it seems to present no great difficulty.

CYNWYD, PA.,
Jan. 25, 1943.

THE THEORY OF CASCADING OF INDUCTION MOTORS MECHANICALLY COUPLED TO A.C. COMMUTATOR MOTORS.

BY

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INTRODUCTION.

Of recent years the problem of speed regulation of induction motors has again come to the fore in specialized applications. In the past many schemes have been suggested and may be broadly segregated into two classes, as:

(1) Those effecting a redistribution of the flux by some rearrangement of the windings. This changes the number of poles.

(2) Those changing the characteristics of the rotor circuit by the use of a passive or of an active network connected across the slip rings.

The first method in its simplest form uses two separate windings arranged for different numbers of poles. As the synchronous speed is given by $n_0 = 2f/p$, changing from one winding to the other varies the synchronous speed of the motor. In a more developed form as exemplified by the Hunt-Creedy windings the windings are cascaded giving $(p_1 \pm p_2)$ poles. This method of speed change gives a definite speed at each step and cannot be described as a complete solution of the problem.

The next method has greater possibilities; initially we consider the passive network in the form of an impedance. On varying the real component the impedance traces out a locus parallel to the axis of the reals (Fig. 1). As this component is increased from zero, the power consumption in the network increases. The power available for work in the rotor is thus decreased. Since we have

$$\text{slip} = \frac{\text{rotor losses}}{\text{rotor input}}$$

it is evident that the speed of the motor is reduced.

On variation of the quadrature component of the network, the locus of the impedance is now parallel to the axis of the imaginary. This has the effect of varying the power factor of the motor without much alteration of speed. As for the ordinary circuit an increase of inductive reactance affects the power factor of the motor adversely, while additional capacity improves it.

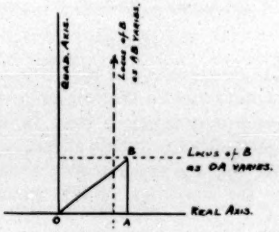


Fig. 1.

This type of network is very common in practice, where resistances are used to reduce the speed and condensers to improve the power factor. Both methods have distinct limitations. The resistance method secures speed regulation at the expense of efficiency, while the condensers are more effective at low speeds and not at the high speeds where correction is most desirable.

Turning now to the active network. Here a voltage is generated. When this voltage is variable in amplitude, phase and frequency conditions of great interest and possibility arise. Due to the necessity of voltage equilibrium in the rotor circuit it is seen that the amplitude and frequency cannot be varied independently. To fulfill this condition an auxiliary machine is used driven by that whose speed requires regulation. Where phase control is specified the auxiliary machine must be of the commutator or the synchronous type.

Comparison of the performance of the active network with that of the passive type reveals that economies are possible. In the latter case the power that is wasted in the resistance of the passive type may be used to produce an additional

V.23
1943

torque either helping the main machine or returning power to the mains. This method of connecting machines is known as cascading.

1. GENERAL DESCRIPTION OF CASCADE CONNECTIONS.

There are two possible methods of connecting machines in cascade. These may be defined as

(i) The mechanically coupled set.

(ii) The electrically coupled set.

These terms need further definition as both sets are electrically connected. In the first method, the rotors are coupled mechanically, while a phase winding on the rotor of the first motor is connected in series with the primary phase of the second motor. Since the machines are mechanically coupled, they must rotate at the same speed. The conditions of commutation on the auxiliary motor if of the commutator type may thus be poor. The second connection avoids this difficulty by removing the mechanical coupling. The power from the slip rings is transmitted to the auxiliary commutator motor which now drives some form of generator which returns power to the supply. This method allows the selection of the most suitable speed for commutation and may be regarded as introducing a further degree of freedom.

On neglecting all losses, the power relations are derived as follows:

Let P be power input to the set, for the mechanically coupled set, the shaft power of the main motor is $(1 - s)P$ and at the slip rings the power is sP. The latter is transmitted to the auxiliary motor and develops therein a shaft power of sP. So the total power is given by $(1 - s)P + sP = P$. The set works at constant power but at variable speed. Thus this connection is referred to as the *constant power* connection.

For the electrically coupled set, the shaft power and slip power are as given above. The slip energy is now transmitted to a separate set and returned to the mains through a generator. The main motor now operates at a variable speed but at a constant torque and so is known as the *constant torque* connection. The theory of sets of this type using neutralized commutator motors has been given by Hull (6).

From consideration of the heating conditions the size of the main motor must be P while that of the auxiliary is sP. The size of the latter thus depends on the desired range of speed.

2. FREQUENCY RELATIONSHIPS.

On considering two motors having p_1 and p_2 poles respectively and being mechanically coupled together, it can be shown that when the main motor has a slip s_1 then the slip of the auxiliary motor is given by

$$s_2 = 1 - \frac{1 - s_1}{s_1} \cdot p_2/p_1$$
$$= 1 + g - g/s_1 \qquad \text{where} \qquad g = p_2/p_1.$$

The value of g depends on how the fields are connected. When the connection is such that both fields rotate in the same direction then g is positive. When the rotation is inverse then g is negative.

The synchronous speed of the set is formulated by $s_2 = 0$ or a speed of $2f/(p_1 + p_2)$ r.p.s. Other useful results may be derived by specifying the slip of the set as s. Then we obtain

$$(1 - s_1)2f/p_1 = (1 - s)2f/(p_1 + p_2)$$

or

$$s_1 = \frac{p_1 + sp_2}{p_1 + p_2}.$$

Likewise

$$s_2 = \frac{(p_1 + p_2)s}{p_1 + sp_2}$$

or

$$s_1 s_2 = s. \qquad (1)$$

Denoting $p_2/(p_1 + p_2)$ by α and $p_1/(p_1 + p_2)$ by β we get

$$s_1 = \alpha + \beta s. \qquad (2)$$

In Fig. 2 these relationships have been plotted for the case where $p_1 = 6$ and $p_2 = 4$.

In the electrically coupled set, the synchronous speed of the commutator motor is $n_2 = 2s_1 f/p_2$. When this motor is coupled to an induction generator of p_3 poles, the synchronous speed thereof is $2f/p_3 = n_3$. Let the commutator motor have

a slip s_2; then we get

$$(1 - s_2)n_2 = n_3$$

or

$$s_2 = 1 - \frac{1}{s_1}\frac{p_2}{p_3}$$

and

$$s_1 = \frac{p_2}{p_3} \cdot \frac{1}{1 - s_2} \cdot \qquad (3)$$

In this case the slip and so the speed of operation is governed by the ratio of the poles (p_2/p_3). With $p_2 = p_3$ we have $s_2 = 1 - \frac{1}{s_1}$. From this equation we see when $s_1 = 1$, $s_2 = 0$. The auxiliary set now runs at synchronous speed at start and

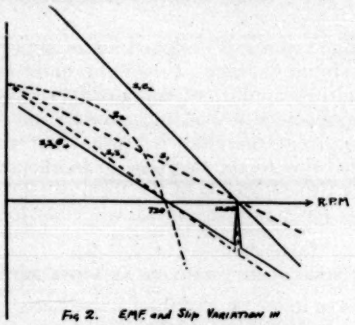


FIG 2. EMF. and Slip Variation in A Mechanically Coupled 6/4 Pole Set.

for values of s_1 less than unity it will run above synchronous speed. When p_2 is less than p_3, at $s_1 = 1$ the set operates below synchronous speed. A decrease of s_1 causes the speed of the auxiliary set to rise. At $s_1 = p_3/p_2$ the speed will be synchronous. When p_2 is greater than p_3 the set will run above synchronous speed at start and its speed will rise with any further increase of s_1. In Fig. 3 the values of s_2 have been plotted for pole ratios of 3/2, 3/3, and 2/3 respectively. As a commutator motor operates best at its synchronous speed, it

is seen that a suitable choice of pole ratio enables operation to occur in that region and so ameliorate commutation conditions.

As in subsequent investigations the question of flux distribution is important this matter is introduced here. Assuming

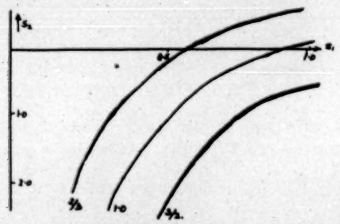


FIG 3. EFFECT OF RATIOS OF POLES ON RELATIVE SLIPS.

that the auxiliary motor is designed for an applied volts equal to the open circuit voltage (V_2) of the main motor, at any slip s, the voltage applied to the auxiliary is given by sV_2. Since the frequency is now sf, normal flux will prevail. The magnetizing current therefore is normal but the iron losses will be reduced due to the decrease of frequency. Since the expression for these losses is in the form of $(As^{1.6} + Bs^2)$ and is too complex for manipulation we will consider the losses to be normal.

V.23
1943

3. AN ELEMENTARY THEORY OF AN ACTIVE NETWORK.

We have seen in the case of the passive network that variation of the real component of the impedance causes a speed change while variation of the quadrature component affects the power factor. In an active net both components are variable and thus the theory of speed control and power factor correction are interlinked. For an induction motor with such a net connected across its slip rings, the following equations express voltage conditions,

$$V = I_1(R_1 + jX_1) + E_1,$$
$$s[E_2 - V_2\epsilon j^\alpha] = (R_2 + jsX_2)I_2.$$

Here $sV_2\epsilon j^\alpha$ denotes the voltage developed at the slip rings by the active net. This voltage will depend on the type of the

second machine. All quantities are phase quantities and have the usual meaning. The current in the rotor is now given by

$$I_2 = \frac{E_2 - V_2\epsilon^{j\alpha}}{R_2/s + jX_2}.$$

If the brush setting as denoted by α is constant, then s is the only variable. So we may write as a general equation

$$x + jy = \frac{a_1 + ja_2}{\lambda b + C_1 + jC_2}.$$

Here λ is the variable. By rationalizing and equating the in phase and quadrature components it is readily seen that the locus of $(x + jy)$ with variation of λ is a circle passing through the origin. The coördinates of the center are at

$$x_0 = a_2/2C_2 \qquad \text{and} \qquad y_0 = -a_1/2C_1.$$

The radius is given by $\frac{1}{2C_2}\sqrt{a_1^2 + a_2^2}$. We now apply this theory to various types of motors.

For the *series* motor, here as in the D.C. machine the applied voltage and current are related as

$$V_2\epsilon^{j\alpha} = (m + jn)I_2.$$

Then we get

$$I_2 = \frac{E_2}{(R_2/s + m) + j(X_2 + n)}.$$

Using the above development we get $x_0 = 0$ and $y_0 = -E_2/2(X_2 + n)$. The radius is $E_2/2(X_2 + n)$. Variation of α would result in a change of m and n, causing the center of

TABLE I.

α	x_0	y_0	Radius
0	0	$-E_2/2(X_2+B)$	$E_2/2(X_2+B)$
$\pi/2$	0	$-E_2/2(X_2+A)$	$E_2/2(X_2+A)$
π	0	$-E_2/2(X_2-B)$	$E_2/2(X_2-B)$
$3\pi/2$	0	$-E_2/2(X_2-A)$	$E_2/2(X_2-A)$
2π	0	$-E_2/2(X_2+B)$	$E_2/2(X_2+B)$

the circle to move along the negative y_1 axis. In Table I the changes for certain various values of α are shown, where we have substituted

$$m = A\cos\alpha - B\sin\alpha \quad \text{and} \quad n = B\cos\alpha + A\sin\alpha.$$

When a *shunt* motor is used, the voltage V_2 is independent of the current and may be written as $V_2(\cos\alpha - j\sin\alpha)$. On substituting in the above equations we get $x_0 = V_2\sin\alpha/(2X_2)$ and $y_0 = -[E_2 - V_2\cos\alpha]/(2X_2)$ with a radius equal to $[V_2^2 + E_2^2 - 2V_2E_2\cos\alpha]^{\frac{1}{2}}/2X_2$. On substituting various values for α the results shown in Table II are obtained.

TABLE II.

α	x_0	y_0	Radius
0	0	$-[E_2 - V_2]/2X_2$	$[E_2 - V_2]/2X_2$
$\pi/2$	$V_2/2X_2$	$-E_2/2X_2$	$\sqrt{[E_2^2 + V_2^2]}/2X_2$
π	0	$-[E_2 + V_2]/2X_2$	$[E_2 + V_2]/2X_2$
$3\pi/2$	$-V_2/2X_2$	$-E_2/2X_2$	$\sqrt{[E_2^2 + V_2^2]}/2X_2$
2π	0	$-[E_2 - V_2]/2X_2$	$[E_2 - V_2]/2X_2$

It will now be observed that the locus of the center is an ellipse whose major axis coincides with the negative y axis (Fig. 4).

When the brush angle is fixed and V_2 varied by a transformer the locus of the center reduces to a circle with center given by $x_0 = 0$ and $y_0 = -2E_2/X_2$. The radius is $V_2/2X_2$.

Further it will be noticed that insertion of $V_2 = 0$ the diagram reduces to that of an induction motor with center at $x_0 = 0$ and $y_0 = -E_2/2X_2$ of radius $E_2/2X_2$.

From considerations of the power in the rotor it may be shown that for the brush angle in the range $\alpha = 0$ to $\pi/2$ the second motor acts as a speed regulator while in the range π to $3\pi/2$ it acts as a phase advancer.

The simple circle derived in the above discussion is not quite in accord with experimental results. This is on account of the impedance of the secondary varying in a complicated manner with the speed. For example the real component of

coupled induction motors has been shown to be (4)

$$R_1 + \frac{R_2' + R_3'}{s_1} + \frac{R_4'}{s_1 s_2},$$

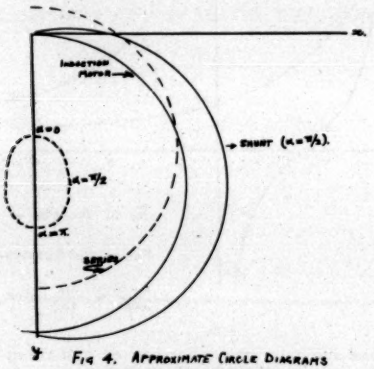


FIG 4. APPROXIMATE CIRCLE DIAGRAMS FOR DIFFERENT TYPES OF CASCADING.

where R_2', R_3' and R_4' denote the resistances of the members referred to the H.T. member. On using the relationships of slip as given in Section 2 the resistance may be written as

$$R_1 + \frac{R_2' + R_3'}{\alpha + \beta s} + \frac{R_4'}{s}.$$

As for an induction motor the slip s is considered to be variable between $+\infty$ and $-\infty$. In Fig. 5 a slip-resistance curve has been plotted for a 6/4 pole combination. The following ranges of resistance occur.

Resistance	Slip
0 to $+\infty$	$+\infty$ to 0
$+\infty$ to $-\infty$	0 to $-\frac{2}{3}$
$-\infty$ to 0	$-\frac{2}{3}$ to $-\infty$

It will be noted for one cycle of slip the resistance term moves through the cycle twice. Thus in Fig. 7*a* the point D will move twice between $+\infty$ and $-\infty$. Since the current is ob-

tained from the impedance diagram by inversion, it follows that the locus of the current will be a reëntrant figure.

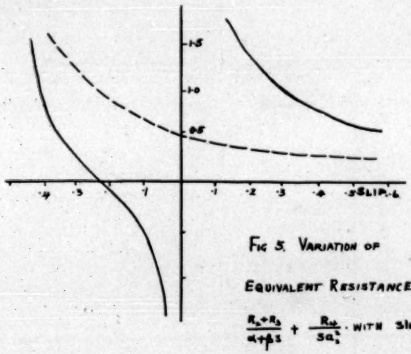


FIG 5. VARIATION OF EQUIVALENT RESISTANCE.

$\frac{R_2+R_3}{a+\beta s}+\frac{R_{22}}{3a_1^2}$ WITH SLIP.

4. MORE DETAILED CONSIDERATION OF CASCADING WITH A SERIES REGULATOR.

We now consider sets employing (1) a series motor and (2) a shunt motor as speed regulator. To separate results it appears advisable to consider the sets individually. In this section the series regulator motor will be discussed. The essential points of the series commutator motor without a rotor transformer will be derived. In all discussions on the polyphase commutator motor the drum connected armature winding is replaced by its equivalent star consisting of $(1/\sqrt{3})$ times the actual turns. Then the equivalent turns per phase on the stator and the rotor are given by T_1 and T_2 respectively. We define the starting position of the brushes as that in which the M.M.F.'s of the stator and rotor are in phase opposition or the brush angle is π. On advancing the brushes by an angle $\pi - \theta$ space degrees from the starting position the M.M.F. diagram is as shown in Fig. 6. From this figure it is seen that the resultant ampere-turns is given by $[\bar{A}_1^2 + \bar{A}_2^2 + 2\bar{A}_1\bar{A}_2 \cos\theta]^{\frac{1}{2}}$. On writing $\frac{T_2}{T_1} = \frac{\bar{A}_2}{\bar{A}_1} = a$ we get that $\bar{A}_R = \bar{A}_1[1 + a\epsilon^{-j\theta}]$. Neglecting saturation effects the

flux may be written as

$$\phi_R = \phi_1[1 + a\epsilon^{-j\theta}],$$

where ϕ_1 is the flux which exists when the stator alone is excited. Alternatively we may use the expression

$$\tan \alpha = \frac{a \sin \theta}{1 + a \cos \theta}$$

and obtain

$$\phi_R = \phi_1[1 + 2a \cos \theta + a^2]^{\frac{1}{2}}\epsilon^{-j\alpha}.$$

This shows that ϕ_R lags on ϕ_1 by an angle α.

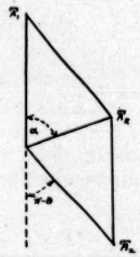


FIG 6 SPACE DIAGRAM OF M.M.F.

The voltage induced (E_1') in the stator winding by the flux ϕ_1 would lag thereon by $\pi/2$ and have an amplitude of $4.44fT_1\phi_1 \times 10^{-8}$. On substituting for ϕ_1 we get

$$\begin{aligned} E_1' &= 4.44fT_1\phi_R\epsilon^{j\alpha}(1 + 2a \cos \theta + a^2)^{-\frac{1}{2}} \\ &= E_1\epsilon^{j\alpha} \quad \text{(say)}. \end{aligned}$$

Thus E_1 lags on E_1' by α and so it lags on ϕ_1 by $(\pi/2 + \alpha)$. In the rotor since the brushes have been advanced by θ, E_2 will lag on ϕ_1 by $(\pi/2 + \alpha - \theta)$. From these the resultant of the induced E.M.F. is obtained as

$$E_1[\epsilon^{-j(\pi/2+\alpha)} + sa\epsilon^{-j(\pi/2+\alpha-\theta)}].$$

The component of the applied voltage to balance this is

given by

$$E_1[(\sin\alpha + sa\sin\overline{\alpha - \theta}) + j(\cos\alpha + sa\cos\overline{\alpha - \theta})].$$

On expressing α in terms of θ we get

$$E_1\left[\frac{a(1-s)\sin\theta + j(1 + sa^2 + \overline{1+s}\, a\cos\theta)}{(1 + 2a\cos\theta + a^2)^{\frac{1}{2}}}\right].$$

Neglecting the magnetizing component of the current the expression for the applied voltage is given by

$$V = I[R_1 + R_2 + j(X_1 + X_2)] + E_1[m + jn],$$

where

$$m = \frac{a(1-s)\sin\theta}{\sqrt{(1 + 2a\cos\theta + a^2)}} \text{ and } n = \frac{1 + sa^2 + \overline{1+s}\, a\cos\theta}{\sqrt{(1 + 2a\cos\theta + a^2)}}.$$

Since E_1 varies as I it is evident that the series motor behaves as a variable impedance.

The watts output are given by KI^2m or on substituting for I we get

$$P = \frac{V^2a(1-s)\sin\theta}{K(1 + 2a\cos\theta + a^2)(1 + 2as\cos\theta + a^2s^2)},$$

where the resistance and reactance of the windings are neglected. Denoting by P_0 the power at $s = 0$ we get

$$P = P_0\frac{1-s}{1 + 2as\cos\theta + a^2s^2}.$$

The torque at any slip is likewise given by

$$T = T_0\frac{1}{1 + 2as\cos\theta + a^2s^2},$$

with a maximum value at $s = \cos\theta/a$. The speed is then given by $n = n_0\left(1 - \frac{\cos\theta}{a}\right)$.

When this motor is cascaded with an induction motor (Fig. 7), let the ratios of the effective turns in the stator and

rotor of the two machines be a_1 and a_2 respectively, the expression for the applied voltage is given by:

$$V = I_1\left[\left(R_1 + \frac{R_2+R_3+R_4}{s_1a_1^2} + \frac{s_1K}{s_1a_1^2}(1-s_2)a_2\sin\theta\right) + j\left(X_1 + \frac{X_2+X_3+X_4}{a_1^2} + \frac{s_1K}{s_1a_1^2}[1+s_2a_2^2+\overline{1+s_2}a_2\cos\theta]\right)\right].$$

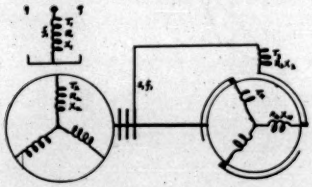


FIG. 7 CASCADE CONNECTION WITH A SERIES REGULATOR.

By the usual method it may be shown that the total output of the set is given by

$$P = \frac{I_1^2}{a_1^2}[(1/s_1 - 1)(R_2 + R_3 + R_4) + K(1 - s_2)a_2\sin\theta].$$

This may be split between the induction motor and commutator motor as follows:

$$P_1 = I_2^2[(R_2 + R_3 + R_4) + s_1K(1 - s_2)a_2\sin\theta]\frac{1 - s_1}{s_1},$$

$$P_2 = I_2^2[s_1(1 - s_2)a_2K\sin\theta].$$

On expressing s_1 and s_2 in terms of s we have, for a given brush setting,

$$\frac{P_1}{P_2} = \frac{1}{\alpha + \beta s}\left[\frac{p_1}{p_2} \times \text{constant} + \beta(1 - s)\right].$$

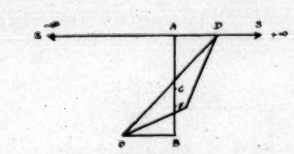
The expressions for the torques in watts are

$$T_1 = \frac{1}{2\pi n_0}\frac{\beta}{\alpha + \beta s}I_2^2[R_2 + R_3 + R_4 + \alpha Ka_2(1 - s)\sin\theta],$$

$$T_2 = \frac{\alpha}{2\pi n_0}I_2^2Ka_2\sin\theta.$$

The impedance diagram of the set is shown in Fig. 8*b*. Here it is advisable to remove s_2 by using the equation

$$s_2 = 1 + \frac{p_2}{p_1}\left(1 - \frac{1}{s_1}\right)$$



$OB = R_1$

$BC = X_1 + \frac{X_2 + X_3}{a_2^2}$

$CA = X_2/a_2^2 a_3^2$.

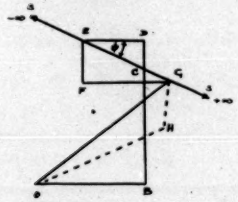
$AD = \frac{R_2 + R_3}{s_1 a_2^2} + \frac{R_3}{s_1 s_2 a_2^2 a_3^2}$

$OD = Z_0$.

OF = Z of main motor.

FD = Z across its Slip Rings.

Fig. 8(a) IMPEDANCE DIAGRAM FOR CASCADED INDUCTION MOTORS.



$OB = R_1$

$BC = X_1 + \frac{X_2 + X_3 + X_4}{a^2}$

$CD = \frac{R_2}{a_2^2}\left[1 + 2a_2\cos\theta + a_2^2\right] + \frac{p_2}{p_1} a_2(a_2 + \cos\theta)\Big]$.

$DE = -\frac{K}{a_2^2} a_2 \frac{p_2}{p_1} \sin\theta$.

$EF = -\frac{1}{s_1}\frac{p_2}{p_1} K a_2(a_2 + \cos\theta)$

$FG = \frac{1}{s_1}\left[\frac{R_2 + R_3 + R_4}{a_2^2} + \frac{p_2}{p_1}\frac{K a_2 \cos\theta}{a_2^2}\right]$

$OG = Z_0$.

OH = Z main motor

HG = Z across Slip Rings.

Fig. (3b) IMPEDANCE DIAGRAM FOR SET WITH SERIES REGULATOR.

and so obtain the secondary impedance of the set as

$$Z_2 = \left[\frac{R}{s_1} + \frac{p_2}{p_1}\left(\frac{1}{s_1} - 1\right) K a_2 \sin\theta\right] + j\left[X + K(1 + 2a_2\cos\theta + a_2^2) + \frac{p_2}{p_1}\left(1 - \frac{1}{s_1}\right) a_2(a_2 + \cos\theta)\right].$$

In Fig. 8*b*,

$$BD = X + K(1 + 2a_2\cos\theta + a_2^2) + \frac{p_2}{p_1} a_2(a_2 + \cos\theta),$$

$$DE = -\frac{p_2}{p_1} K a_2 \sin\theta.$$

The vector OD is a constant for any given brush position. It will be noted that the other components give a line whose slope, i.e., $\tan \phi$, is indpendent of slip. Thus the point G must lie on a line as EG produced.

It is seen when $\cos \theta = -a_2$ that EG is parallel to the axis of the reals and the expression for the impedance of the secondary reduces to the form $(R/s + jX)$ which is that for an induction motor. Under this condition the speed-load curve has a drooping characteristic. On the other hand if $\sin \theta = -R \frac{p_1}{p_2} \cdot \frac{1}{Ka_2}$ then EG is parallel to the quadrature axis. Here the impedance of the secondary reduces to $(R + jX/s)$. In this instance the second machine behaves as a phase advancer. In other words the resistance or the reactance of the regulator circuit may be varied. Of these, the former is of more interest here. There are three possibilities, namely, making CD greater than, equal to, or less than AB, i.e., making the effective resistance positive, zero, or negative. From induction motor theory we have

$$\text{slip} = \frac{\text{rotor copper loss}}{\text{rotor input}}.$$

Thus with a positive resistance, the slip increases with increasing load, and the load-speed characteristic is that of a cumulatively compounded D.C. motor. With a negative resistance, the slip will be negative and the speed of the motor increases as for a differentially compounded D.C. motor. When the resistance is zero, the motor would tend to behave as a synchronous motor.

From this discussion it is evident that by attention to design particulars of the series regulator the motor speed characteristics may be varied quite widely. It will also be observed since a series commutator motor develops torque at its synchronous speed there is no discontinuity in the speed-torque curve of such a cascaded set as is the case with induction motors connected in cascade.

5. CASCADE WITH A SHUNT REGULATOR.

We now turn to the case of a set with a shunt commutator motor. The salient points in the theory of the shunt motor

will be developed first. Here a motor whose rotor is fed through a transformer of ratio K is considered. The connection diagram is shown in Fig. 9. The voltage on the secondary

FIG 9. DIAGRAM OF SHUNT MOTOR WITH ROTOR TRANSF...ER.

side of the transformer is KV or neglecting the stator impedance drop is Ke_3 where e_3 is the voltage induced in a phase of the stator winding. Due to the brush shift the voltage injected into the rotor will be advanced θ and the current in the rotor when reflected into the stator must be retarded by θ (Fig. 10). The total voltage in the rotor is thus given by

$$e_4 = s_2 e_3 + K e_2 \epsilon^{j(\pi-\theta)}.$$

Denoting the ratio of transformation $t_3/t_4 = 1/a_2$, we get

$$\begin{aligned} e_4 &= e_3\left(s_2 - \frac{K}{a_2}\epsilon^{-j\theta}\right) \\ &= e_3(s_2 - m\epsilon^{-j\theta}) \qquad \text{with} \qquad m = K/a_2 \\ &= e_3\sqrt{(s_2^2 - 2ms_2\cos\theta + m^2)}\cdot\epsilon^{j\psi_2}, \end{aligned}$$

where

$$\tan\psi_2 = m\sin\theta/(s_2 - m\cos\theta).$$

The rotor current is thus, $i_4 = e_4/Z_4$, so that

$$|I_2| = \frac{|e_3|}{\sqrt{R_4^2 + X_4^2}}\sqrt{(s_2^2 - 2ms_2\cos\theta + m^2)},$$

and

$$\cos\phi_2 = \cos(\psi_2 - \alpha) = \frac{s_2\cos\alpha - m\cos\overline{\theta + \alpha}}{A},$$

where

$$\tan \alpha = X_4/R_4 \qquad \text{and} \qquad A = (s_2^2 - 2ms_2 \cos\theta + m^2)^{\frac{1}{2}}.$$

The current in the stator to balance this load current is

$$-a_2 i_4 = -\frac{a_2 e_3}{Z_4}(s_2 - me^{-j\theta}).$$

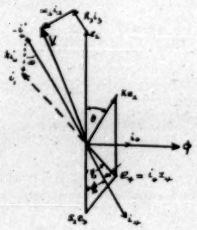


FIG 10. VECTOR DIAGRAM OF SHUNT MOTOR.

Neglecting the magnetizing current we have the applied voltage given by

$$V = I_3 Z_3 + e_3/a_2$$
$$= I_3\left[R_3 + jX_3 + \frac{(R_4 + jX_4)(s_2 - m\cos\theta - jm\sin\theta)}{a_2^2 A^2}\right];$$

so the power input is $I_3^2\left[R_3 + \frac{R_4}{a_2^2 A^2}\left(s_2 - m\frac{\cos\overline{\theta + \alpha}}{\cos\alpha}\right)\right]$ and the input to the rotor is $I_4^2 R_4 \frac{\cos\overline{\psi_2 - \alpha}}{A\cos\alpha}$. On subtracting the copper loss in the rotor winding $(i_4 e_4 \cos\alpha)$ and the input through the transformer $Ke_3 i_4 \cos\overline{\theta + \alpha}$ the output of the motor is obtained as

$$(1 - s_2)e_3 i_4 \cos\overline{\psi_2 - \alpha} = (1 - s_2)\frac{i_4^2 R_4}{A}\frac{\cos\phi_2}{\cos\alpha}.$$

On denoting the synchronous speed by n_0 the torque in watts per phase is obtained as

$$\frac{e_3 i_4 \cos \overline{\psi_2 - \alpha}}{2\pi n_0} = \frac{1}{2\pi n_0} \frac{e_3^2}{a_2^2(R_4^2 + X_4^2)} \left[s_2 - m \frac{\cos \overline{\theta + \alpha}}{\cos \alpha} \right].$$

The no load slip is given by $(m \sec \alpha \cos \overline{\theta + \alpha})$ and the no load speed by $n_0[1 - m \sec \alpha \cos \overline{\theta + \alpha}]$. From these expressions it is seen that the equivalent resistance of the secondary is $\frac{R_4}{a_2^2 A^2} \left(s_2 - m \frac{\cos \overline{\theta + \alpha}}{\cos \alpha} \right) = \frac{R_4}{a_2^2 A} \cdot \frac{\cos \phi_2}{\cos \alpha}$ and the equivalent reactance is

$$\frac{R_4}{a_2^2 A^2} \left(s_2 \tan \alpha - m \frac{\sin \overline{\theta + \alpha}}{\cos \alpha} \right) = - \frac{R_4}{a_2^2 A} \frac{\sin \phi_2}{\cos \alpha}.$$

When this motor is connected in cascade with an induction motor (Fig. 11), on denoting $\frac{e_1}{e_2} = \frac{i_2}{i_1} = \frac{1}{a_1}$ and $\frac{e_3}{e_4} = \frac{i_4}{i_3} = \frac{1}{a_2}$,

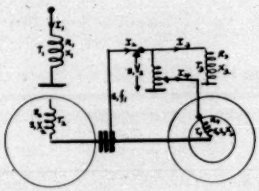


FIG. 11. SET WITH SHUNT REGULATOR.

we get

$$V = I_1 \left[\left(R_1 + \frac{R_2}{s_1 a_1^2} \right) + j \left(X_1 + \frac{X_2}{a_1^2} \right) \right] + \frac{V_2}{s_1 a_1^2},$$

where V_2 is the voltage across the terminals of the shunt motor. Using the value of V_2 given above we have

$$V = I_1 \left[\left(R_1 + \frac{R_2}{s_1 a_1^2} \right) + j \left(X_1 + \frac{X_2}{a_1^2} \right) + \frac{1}{s_1 a_1^2} \left\{ \left(R_3 + \frac{R_4}{a_2^2 A} \cdot \frac{\cos \phi_2}{\cos \alpha} \right) + j \left(s_1 X_3 - \frac{R_4}{a_2^2 A} \frac{\sin \phi_2}{\cos \alpha} \right) \right\} \right].$$

We obtain the input to the set as

$$I_1^2\left[R_1 + \frac{R_2 + R_3}{s_1 a_1^2} + \frac{1}{s_1 a_1^2} \cdot \frac{R_4}{A a_2^2} \frac{\cos \phi_2}{\cos \alpha}\right].$$

On subtracting the losses in the induction motor and those of the commutator motor the output of the set is obtained as

$$I_2^2\left[(R_2 + R_3)\left(\frac{1}{s_1} - 1\right) + \frac{R_4}{s_1 a_2^2 A} \frac{\cos \phi_2}{\cos \alpha}\right] - s_2 e_3 i_4 \cos \phi_2.$$

The latter term gives the losses in the armature of the commutator motor and may be reduced to $s_2 I_4^2 \frac{R_4}{A} \frac{\cos \phi_2}{\cos \alpha}$. The outputs of the induction motor and commutator motor are as follows:

$$P_1 = \left(\frac{1}{s_1} - 1\right) I_2^2\left[R_2 + R_3 + R_4 \frac{\cos \phi_2}{a_2^2 A \cos \alpha}\right]$$

and

$$P_2 = (1 - s_2) I_2^2 R_4 \frac{\cos \phi_2}{a_2^2 A \cos \alpha}.$$

The torques are given by

$$T_1 = \frac{\beta}{\alpha + \beta s} \frac{I_2^2}{2\pi n_0}\left[R_2 + R_3 + R_4 \frac{\cos \phi_2}{a_2^2 A \cos \alpha}\right] \text{watts},$$

$$T_2 = \frac{\alpha}{\alpha + \beta s} \frac{I_2^2}{2\pi n_0} R_4 \frac{\cos \phi_2}{a_2^2 A \cos \alpha} \text{ watts},$$

where n_0 is the synchronous speed.

The impedance diagram of this set is shown in Fig. 12, wherein CD represents the impedance across the slip rings and will be given in the form $V_2 \epsilon^{-i\theta}/(I_1 s_1 a_1^2)$. For a fixed amplitude the impedance CD will trace out a circle as the brush angle is varied.

From the expression for the impedance it is seen on making $s_1 X_3 - \frac{R_4}{a_2^2 A} \frac{\sin \phi_2}{\cos \alpha} = 0$ that the shunt regulator is operating at unity power factor. On making this expression negative the regulator will operate as a phase advancer improving the power factor of the set.

It will be noted that the power factor of the armature is given by

$$\cos \phi_2 = \frac{s_2 \cos \alpha - m \cos \overline{\theta + \alpha}}{\sqrt{(s_2^2 - 2ms_2 \cos \theta + m^2)}}.$$

On making $\theta = 0$ or π this reduces to $\cos \phi_2 = \cos \alpha$, i.e., the power factor of the armature is fixed by the constants of the circuit. At any other angle this does not apply.

OB = R_1,
AB = $X_1 + X_2/a_2^2$
AC = R_2/sA_2^2
CD = $\frac{E_1}{E_2 A_2} \cdot \frac{1}{X_2}$
OD = Z (TOTAL).
OC = IMPEDANCE OF INDUCTION MOTOR.
CD = IMPEDANCE ACROSS SLIP RINGS.

FIG 12. APPROXIMATE VECTOR DIAGRAM WITH SHUNT REGULATOR.

From the expression for the impedance we have the real component as $R_3 + \frac{R_4}{a_2^2 A} \frac{\cos \phi_2}{\cos \alpha}$. On substituting $\theta = 0$ this expression becomes $R_3 + \frac{R_4}{a_2^2} \cdot \frac{1}{s_2 - m}$. According as s_2 is greater or less than m the resistance effect is increased or reduced. With $\theta = \pi$ the resistance reduces to $R_3 + \frac{R_4}{a_2^2} \cdot \frac{1}{s_2 + m}$. From these two particular values of θ it is seen that the resistance across the slip rings of the induction motor is variable occasioning a speed change. This discussion shows that shunt regulation is not as flexible as the series type. In fact the shunt regulator has similar characteristics as the induction coupled set, over which an advantage in point of power factor and of operation on several speed ranges depending on the voltage injected into the rotor by the transformer may be claimed.

CONCLUSION.

An approximate theory of cascading with a series and a shunt type of regulator has been developed. The important case using a frequency converter has not been discussed. Here the power is not converted into torque but is fed back into the line. Circle diagrams obtained on cascading standard laboratory machines are shown in Fig. 13. These machines

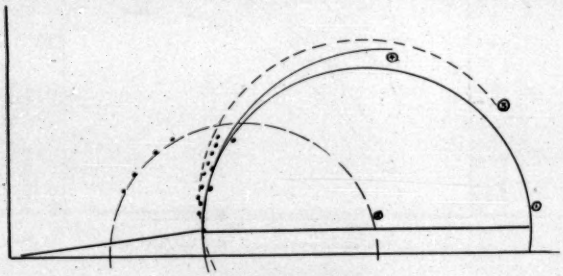


FIG 13 RELATIVE CIRCLES FOR CASCADING.
(I) INDUCTION
II SERIES REGULATOR
III SHUNT REGULATOR
IV COMMUTATOR MOTOR WITH S.C. BRUSHES.

comprise a 10 H.P. 6 pole induction motor with an 8 H.P. 4-pole commutator machine. This machine was not of the neutralized type and could be connected either as an induction type, a series or a shunt commutator type. The usual test results are shown in Fig. 14*a*, *b*, *c* and *d* respectively. It will be noted that in every type the load-speed characteristic is that of the shunt motor.

In all sets the maximum output of the main motor is reduced, which is the sacrifice that must be made to obtain the benefit of speed control. The mechanically coupled set has the objection that the frequency applied to the commutator rises with fall in speed of the set. The flux conditions are constant neglecting the effect of saturation. However the question of design of the commutator limits the high speed while at low speed the commutator is costly due to the large dimensions required. In practice, these strictures have been removed by using the electrically coupled sets.

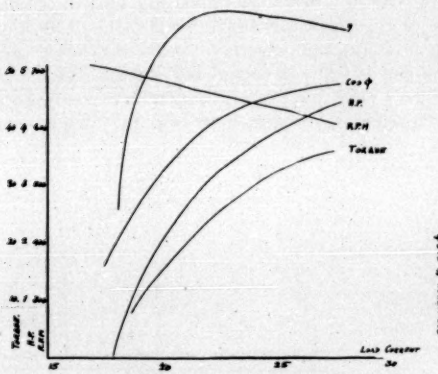


FIG 14(a) CASCADE WITH INDUCTION CASCADE.

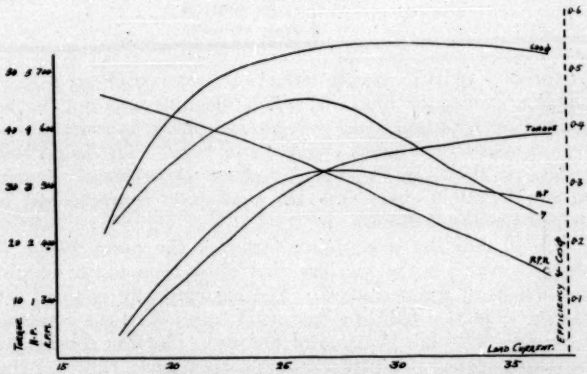


FIG 14(b) CASCADE WITH COMMUTATOR MOTOR BRUSHES SHORT CIRCUITED.

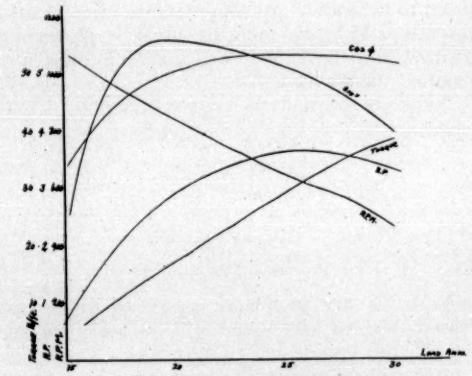


Fig 14(a) Cascade with Series Regulator.

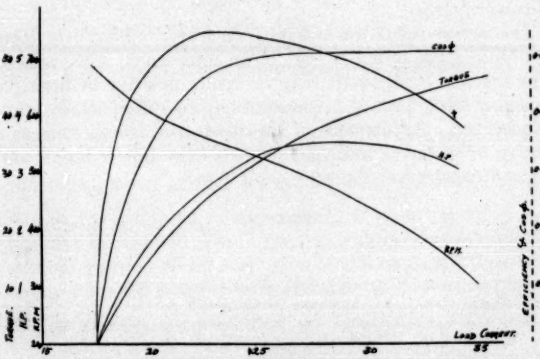


Fig 14(b) Cascade with Shunt Regulator.

For the neutralized shunt regulator the armature E.M.F. is proportional to the speed. At some definite adjustments of the brushes this E.M.F. (e_2) may be either in phase or in phase opposition, with the E.M.F. (e_1) at the slip rings of the induction motor. So we have

$$se_1 = \pm (1 - s)e_2,$$

or

$$s = \frac{e_2}{e_1 \pm e_2},$$

and

$$1 - s = \frac{e_1}{e_1 \pm e_2} = \frac{\text{speed}}{\text{synchronous speed}}.$$

If the E.M.F.'s are in phase opposition below synchronous speed, the no load speed will be less than synchronous, but if they oppose one another above synchronous speed the speed will be greater than synchronous. At any other adjustment of the brushes there will be an in-phase component of the armature voltage which will produce a variation of speed while a quadrature component will compensate for power factor.

The series regulator behaves somewhat differently from the above. Its action is that of an impedance. By adjustment the real component may be either positive or negative producing the effect of a cumulatively or differentially compounded set. Adjustment of the quadrature component as a positive or negative reactance controls the power factor and overload capacity of the set.

REFERENCES.

(1) SPECHT, A.I.E.E., Vol. 27, 1908, p. 791.
(2) MEYER & SYKES, A.I.E.E., Vol. 31 (2), 1912, p. 2067.
(3) CHAPMAN (book), Induction Motors, Wiley & Sons, p. 178.
(4) LANGSDORF (book), A.C. Machines, McGraw-Hill, p. 636.
(5) ARNOLD & LACOUR (book), Die Wechselstromkommutatoren Maschinen (1912). Gives considerable information on this class of machine.
(6) HULL, A.I.E.E., Vol. 39, Part 2, 1920, p. 1135.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

STANDARD SAMPLES OF HYDROCARBONS.

At the urgent request of several organizations, chief of which is the Petroleum Industry War Council, through its Technical Advisory Committee, plans have been prepared for the provision by the Bureau of standard samples of hydrocarbons of a known high purity, for use in calibrating the following apparatus and instruments used in the petroleum, rubber, and allied industries:

(*a*) Mass spectrometers.
(*b*) Infrared spectrographs.
(*c*) Ultraviolet spectrographs.
(*d*) Raman spectrographs.
(*e*) Refractometers.
(*f*) Pycnometers and specific gravity balances.
(*g*) Boiling-point apparatus.
(*h*) Freezing-point apparatus.
(*i*) Molecular-weight apparatus.
(*j*) Bomb-combustion calorimeter for volatile liquids.

Actual operations are scheduled to begin July 1, 1943.

TENSILE CRACKS IN AXIALLY REINFORCED CONCRETE CYLINDERS.

In many instances, design tensile stresses for steel reinforcement in concrete are limited to values considerably below those warranted by strength considerations. The lower stresses are used when it is feared that higher values would result in cracks wide enough to cause leakage, corrosion, or unsightliness. Reduction of design tensile stresses is one method for minimizing the widths of cracks, but it is not the only one. A study of the factors that control the spacing and width of tensile cracks in symmetrically reinforced concrete members was completed recently by D. Watstein and

* Communicated by the Director.

D. E. Parsons, and the results are given in the July Journal of Research (RP 1545).

Concrete cylinders of different sizes and made with two concretes of widely different strengths, were cast with either one or two steel reinforcement bars along their axes. The reinforcement consisted of plain round, deformed, webbed Isteg, and threaded bars. These specimens were tested by applying tensile loads to the reinforcement which protruded from the ends of the cylinders. Observations were made of elongation of the concrete and the spacing and width of tensile cracks.

The factors governing the spacing and width of cracks in symmetrically reinforced concrete members subjected to axial forces were analyzed theoretically. The equations developed and the test data agreed well with respect to the effect of the principal factors, but the values of numerical coefficients which, according to theory, depend only on the distribution of bond stresses, were markedly affected by changes in the shape of the reinforcement bars. According to theory, the spacing of cracks is proportional to the ratio of the diameter of the reinforcement to the percentage of reinforcement in the concrete. It is also proportional to a coefficient which depends upon the shape of the reinforcement. The width of cracks is proportional to the spacing and to the difference between the tensile stress in the reinforcement at a crack and a hypothetical stress called for convenience "stress at zero width of crack."

The results of the investigation indicate that a reduction of widths of cracks, as well as more economical use of steel reinforcement, would be likely to result from the development and general use of an improved type of deformed reinforcement bar for concrete.

ROOFING OF ARMY BUILDINGS

The Bureau has coöperated closely with the Repairs and Utilities Branch, Construction Division, Office of the Chief of Engineers, War Department, on problems connected with the maintenance of roofs and the reroofing of Army buildings.

The total area of the roofs of Army buildings is approximately 11,000,000 squares, equivalent to 39.5 square miles. These buildings were erected and their roofs applied with all possible speed. Of necessity, unseasoned lumber was frequently used for roof sheathing, and in many cases the roofs were applied at temperatures below freezing by inexperienced workmen. As a consequence, roofing has been one of the major problems of the Repairs and Utilities Branch, and promises to become more serious with time.

Following inspections of cantonment roofings in the Third and Fourth Service Commands early in 1942, the Bureau prepared the section on roofing of the Repairs and Utilities Manual, outlining methods for the repair and maintenance of all types of roofs and for reroofing when necessary.

Further inspections were made in the Eighth and Ninth Service Commands during last January and February. As a result of these inspections, it was estimated that about 10 per cent. of the total roof area will require reroofing during the fiscal year 1944 at a cost of approximately $10,000,000. It was also estimated that 45 per cent. of the total area will require some repair during the same period, at a cost of $5,500,000.

Following these inspections, additional instructions were prepared by the Bureau, outlining the conditions where reroofing is indicated, and giving further detailed methods of maintenance and repair. These have been sent to the Service Commands for distribution to all stations.

Every effort has been made to keep reroofing at a minimum, because it is realized that a general reroofing program would not only be uneconomical at present but would probably overtax the facilities of manufacturers as well as those of roofing contractors. In order to acquaint the personnel of each Service Command with the conditions that may be met in the field and to present the over-all picture of the Army roofing problems it has been proposed to have a representative of the Bureau visit several stations in each Service Command in company with the engineer who processes requests for roof repairs and reroofing.

LOW-TIN SOLDERS

In response to the current need for information on the utility of soft solders containing 20 per cent. or less of tin, as required by a recent WPB order, a practical study of the solders included in recent Federal and SAE Specifications is being conducted at the Bureau. A report by J. A. Kies and W. F. Roeser covering only the first phases of the investigation was presented at the annual meeting of the American Society for Testing Materials last month.

Six solders containing 20 to 50 per cent. tin, small additions of antimony, bismuth, and copper, and the remainder lead were studied. One of the 20-per cent.-tin solders contained 1.25 per cent. silver. Spreading tests on copper showed the higher tin solders to best advantage, an advantage not maintained on black iron. Strength tests of simple lap joints in sheet metals proved that for some practical applications the low tin solders are just as strong as the old stand-bys. The main disadvantage of the low-tin solders is the relative difficulty in handling. The thickness of solder film for maximum strength of joint is no greater for a 20-per cent.-tin solder than for 50–50 solder.

ELECTROPLATING OF MILITARY SUPPLIES.

The scarcity of metals such as copper, zinc, nickel, aluminum, and their alloys has led to the substitution of steel for many military supplies formerly made of these nonferrous metals. In most cases, the steel articles require protection against corrosion, which is frequently accomplished with electroplated coatings.

According to Dr. William Blum who delivered a paper entitled "Applications of Electroplating to Military Supplies" at the meeting of the American Electroplaters' Society in Buffalo on June 7, relatively little nickel plating is now applied. Cadmium plating is extensively employed on fuse parts and airplane fittings, but zinc is substituted wherever possible. Lead plating is being used to an increasing extent, and chromium and silver coatings find many applications.

Camp tests on plated steel tableware show that resistance to abrasion is more important than protection against corrosion. Chromium coatings directly on case-hardened steel are useful on both tableware and mess trays.

The electroplating of military equipment has absorbed the full capacity of most of the plants formerly engaged in plating civilian supplies.

USE OF SHORE DUROMETER FOR MEASURING HARDNESS OF RUBBER.

Two recent developments have focussed attention on the importance of the time factor in using the Shore Durometer for measuring the hardness of rubber. On the one hand, most of the synthetic rubbers now coming into large scale use exhibit considerably more creep than natural rubber, and this creep greatly increases the probability of obtaining large variations in the Durometer reading when the test is made by present methods. On the other hand, the Durometer reading has been made a requirement of many purchase specifications so that, whereas it was once regarded as an approximate measurement, it has now become a basis for the acceptance or rejection of large deliveries of rubber goods.

A paper describing tests made at the Bureau to show the effect of time on the Durometer readings was presented at the 46th annual meeting of the American Society for Testing Materials on June 28, by Rolla H. Taylor.

In this investigation the personal equation was eliminated by the use of mechanical means for applying the Durometer. Motion pictures of the instrument and a stop watch in the same field of view made possible a detailed study of the change in Durometer reading with time. Tests were made on 16 specimens representing 5 different types of synthetic rubber and two kinds of natural rubber, designed to cover the range of hardness for which the Shore Durometer is commonly used.

Curves drawn to represent the Durometer readings as a function of time all exhibit a rapid decrease of the reading during the first few seconds with progressively less change for succeeding time intervals. The general form of the curve is the same for all compositions but the amount of the decrease

varies considerably and is in general much greater for synthetic than for natural rubber.

While these curves indicate the magnitude of the creep, they do not lend themselves well to evaluating the creep quantitatively. If, however, the Durometer readings are plotted against the logarithm of the time, curves are obtained which approximate straight lines, and their slopes may be taken as a measure of creep. These slopes can also be determined approximately from readings at two time intervals, for instance, 3 and 30 seconds, or 5 and 50 seconds.

Measurements were made at three different temperatures and it was found that some specimens, particularly those from synthetic rubber, exhibited a sufficient change to require a specification of the temperature.

The following procedure was recommended when making hardness measurements with the Durometer:

(1) Hold the Durometer in the recommended manner in the right hand with the indentor just touching the specimen and with the right wrist or forearm resting on the specimen or its support. Hold a watch in the left hand.

(2) Make several trial applications to determine the minimum force required to seat the Durometer on the specimen.

(3) At the desired time apply the Durometer as quickly as possible with the minimum required pressure; be careful to maintain constant pressure throughout the test and to hold the Durometer motionless.

(4) Take the reading 30 ± 1 second after the application of the Durometer.

TABLES OF PLANCK'S RADIATION FUNCTONS, AND ELECTRONIC FUNCTIONS.

Mathematical Table MT17, prepared by the Work Projects Administration for the City of New York under the sponsorship of the Bureau, is in two parts, dealing with Planck's radiation functions and the electronic functions, respectively.

Part 1, which was originally published in the *Journal of the Optical Society of America* for February 1940, evaluates

(in Tables I to IV) the functions:

$$R_\lambda = c_1\lambda^{-5}(e^{c_2/\lambda T} - 1)^{-1};\ R_{0-\lambda} = \int_0^\lambda R_\lambda d\lambda,$$

$$N_\lambda = 2\pi c\lambda^{-4}(e^{c_2/\lambda T} - 1)^{-1};\ N_{0-\lambda} = \int_0^\lambda N_\lambda d\lambda.$$

$R_\lambda d\lambda$ is the energy emitted by a black body at absolute temperature, T, on the Kelvin scale, per unit time, per unit area, in a wave-length interval from λ to $\lambda + d\lambda$, throughout the solid angle 2π steradians.

$N_\lambda d\lambda$ represents the number of photons emitted in the wave-length range from λ to $\lambda + d\lambda$, per unit time, per unit area, throughout the solid angle 2π steradians.

The tables evaluate these four functions of λT not only relative to their maximum values, but also for $T = 1000°$ K. in absolute units (ergs or photons per cm.^{2} per second). The values are given to five significant figures, first and second differences being provided to assist in finding R_λ and N_λ by interpolation; a convenient method of correcting for small changes in the value of c_2 from that ($c_2 = 1.436$ cm. K.°) used in computing the tables is included. The function N_λ is also evaluated for $T = 1500$, 2000, 2500, 3000, 3500, and 6000° K.

Part 2, a table of 28 pages, gives in five columns the values for the velocity of an electron relative to the velocity of light, β; $G = 1/(1 - \beta^2)^{\frac{1}{2}}$; G; the energy in electron kilovolts, and the curvature of the electron path in a magnetic field times the field $H\rho$. The total energy, effective mass, and momentum depend on the functions G and βG. The energies cover the range from 6 to 10^{10} electron volts so that the tables are applicable in the full range of energies involved in nuclear and cosmic ray physics. In the range from 10,000 to 1.5×10^7 electron volts successive values differ by less than 1 per cent. in energy and curvature so that in nearly all practical problems the tables would be used without interpolation. As relativistic equations must be used throughout this range the tables will save a great deal of laborious computation.

Copies of MT17 are obtainable from the National Bureau of Standards, Washington, D. C. The price is $1.50.

Glass For Precision Gages.—COL. H. B. HAMBLETON. (*Mechanical Engineering*, Vol. 65, No. 4.) In an effort to contribute to the program for the conservation of steel, the Ordnance Department has initiated a development project of glass gages. The responsibility for the development project has been assigned to the Frankford Arsenal. This arsenal, with the aid of the most progressive glass manufacturers in the country, has formulated tentative specifications and provisional standard drawings of glass gages. These standards, in so far as possible, will be dimensionally the same as those of the American gage design standard shown in publication CS8–43 of the National Bureau of Standards. The arsenal has procured from the glass manufacturers a limited number of glass gages for experimental and development purposes. One of these gages has been used in the cartridge-case shop for the inspection of 57 mm. cartridge cases. This gage performed 260,000 gaging operations before it was worn out. For the first 160,000 operations the wear was only 0.00005 in. A steel gage for this same duty normally has a life of 60 to 70 thousand operations. The Ordnance Department now has several glass gages under test on brass gears for mechanical time fuzes, steel parts for other fuzes, and cartridge case inspection. The results indicate that glass gages are giving very satisfactory performance and, in some cases, show very definite advantages over steel gages some of which are as follows: (1) Many greasings and degreasings are eliminated, since no question of rust is involved. (2) Glass gages are much easier in handling inasmuch as they are much lighter than steel. (3) Glass gages afford visibility in inspection which is not always possible with steel. (4) Glass will more or less teach the inspectors to have respect for handling gages. (5) Perspiration on the hands of the inspectors has no corrosive effect on glass. (6) When the component is very near the size of the gage, there is less tendency for the component to seize or gall in or on the gage.

R. H. O.

V.2
194

THE FRANKLIN INSTITUTE.

MEDAL DAY PROCEEDINGS

The Dinner Meeting and Presentation of Awards on Medal Day at The Franklin Institute convened in Franklin Hall at five-thirty o'clock on Wednesday evening, April 21, 1943, Mr. Charles S. Redding, President of The Franklin Institute, presiding.

Reception to Medalists.

The exercises began with a reception to the Medalists who were to be honored later in the evening. It was attended by four hundred and fifty guests and was given by the Hostess Committee of the Institute, whose officials and members formed a receiving line. At six, dinner for Medalists and guests was served in Franklin Hall.

At eight the meeting was called to order by Mr. Redding, who introduced Mrs. Edith E. Braun, Curtis Institute of Music, who had conducted musical programs at The Franklin Institute during the past year, in the absence in service of Guy Marriner, well known Institute musician.

After the playing of the National Anthem by Mrs. Braun, a toast (in water) was drunk to Benjamin Franklin.

Stated Monthly Meeting.

The President announced that this was a stated monthly meeting of The Franklin Institute; and further, that the Minutes of the last meeting were printed in full in the April Journal. He then asked for corrections and as none were offered, upon motion, the Minutes were approved as printed.

Mr. Redding then addressed the assembly:

"Medal Day is a happy occasion; it is really the high point in The Franklin Institute year. We always have a fine group of people on this occasion to honor those upon whom the Institute would bestow its awards. Tonight we have with

us what I have every reason to believe is a record attendance. Never before in the history of this Institute, to my knowledge, and to the knowledge of those who know the records better than I do, have we had so many in attendance at Medal Day.

"On behalf of the Board of Managers it is my pleasure to welcome you all here tonight. In spite of 'points' and 'rationing,' we have had a satisfying dinner, and we are now prepared for the exercises of the evening.

"Those of you who are familiar with the Medal Day proceedings have noted several changes. In previous years it has been our custom for a long time to have an afternoon session at which the formal Medal presentations were made, and a quite formal dinner meeting in the evening in honor of those who in the afternoon had received those Medals. This year, because of war conditions and difficulties in travel, it was thought wise to combine the two sessions into one and to have this informal evening meeting at which the medals would be presented and at which we would have opportunity to greet the Medalists.

WORK OF COMMITTEE ON SCIENCE AND THE ARTS.

"This is the time the work of the Committee on Science and the Arts of the Institute really becomes apparent. Since the start of the Institute in 1824, it has been conferring medals, but the Committee on Science and the Arts is quite young. It was formed ten years after the start of the Institute, in 1834—that is 109 years ago—and has been continually operating ever since.

"Particularly to our new Medalists and to members and friends of the Institute who are for the first time attending a Medal Day ceremony, a word about the Committee on Science and the Arts may not be out of place. The Committee consists of approximately sixty men who study new developments in science and the arts with a view of selecting individuals who are outstanding in their contributions to world progress, particularly along scientific lines.

"Many suggestions come to this Committee, for study; those considered to be meritorious are referred to appropriate sub-committees for further intensive studies. These sub-committees search records, conduct correspondence, examine

models and handle endless detail, leading to intelligent and considered opinion as to whether a particular development is worthy of the recommendation that an award be made. If the recommendation of the sub-committee is favorable, it is sent to the main Committee and is then turned over to the Board of Managers of the Institute, and if approved, the recipient is so notified and presents himself on Medal Day, here, at the Institute, and the award is made. I say all this in order that you may see that our Medalists are carefully considered, that it is a real honor to receive a Franklin Institute award. These are recognized and honored all over the world.

SOUTH AMERICAN SCIENTIST HONORED.

"We are here to honor these men who have come from various parts of the country to receive deserved recognition.

"As is often the case, there is included in the list of Medalists a scientist from a foreign country—this time, Peru. It is fitting in these days, when our Good Neighbor Policy is being stressed, that one of our awards should go to South America. The recipient is not with us but is represented by an official of his country who will later be introduced and will receive the Medal for him.

"I have a telegram received today from Senor Ballén, the gentleman from Peru who is to be awarded the Potts Medal. It is addressed to Henry Allen, Franklin Institute, Philadelphia, Pa. This came to us in Spanish and was translated this afternoon. 'I beg to express again to the Board of Managers and to the distinguished members of The Franklin Institute, and in particular to the members of the Committee on Science and the Arts, my deep appreciation of the high honor which they have had the kindness to award to me this year—the Howard Potts Medal; and to assure them that urgent reasons prevent me participating in this exceptional distinction to be conferred on me by their erudite Institute; and to express my profound felicitations.'

INTRODUCTION OF MEDALISTS OF FORMER YEARS.

"Tonight, we have with us as our guests a number of those who have received awards in previous years, and I am glad

of the opportunity of presenting them now. I am going to call out the names and I ask the gentlemen to rise. The list is in the chronological order in which the awards were made:

1904	Cresson Medal	Dr. G. H. Clamer, Philadelphia
1906	Longstreth Medal	Dr. G. H. Meeker, Philadelphia
1911	Longstreth Medal	Dr. Joseph S. Hepburn, Philadelphia
1921	Certificate of Merit	
1918	Longstreth Medal	Dr. H. Jermain Creighton, Swarthmore College
1939	Potts Medal	
1920	Longstreth Medal	Mr. G. L. Kothny, Strafford, Pa.
1921	Longstreth Medal	Dr. L. H. Adams, Bethesda, Md.
1922	Potts Medal	Mr. C. Raymond Downs, Old Greenwich, Conn.
1923	Cresson Medal	Mr. Albert W. Kingsbury, Greenwich, Conn.
1923	Longstreth Medal	Mr. Harry S. Parks, Pottstown, Pa.
1924	Longstreth Medal	Mr. Thomas C. McBride, Philadelphia, Pa.
1924	Longstreth Medal	Mr. M. Roy Sheen, Philadelphia, Pa.
1925	Longstreth Medal	Mr. William E. Hoke Baltimore, Md.
1936	Longstreth Medal	
1928	Longstreth Medal	Dr. Frank N. Speller, Pittsburgh, Pa.
1930	Wetherill Medal	Mr. Charles S. Chrisman, West Chester, Pa.
1930	Cresson Medal	Dr. Irving E. Moultrop, Belmont, Mass.
1932	Wetherill Medal	Dr. Frank Wenner, National Bureau of Standards
1934	Cresson Medal	Mr. Stuart Ballantine, Boonton, New Jersey
1935	Wetherill Medal	Dr. James E. Shrader, Drexel Institute of Technology
1936	Longstreth Medal	Mr. Peter P-G. Hall, Philadelphia, Pa.
1938	Longstreth Medal	Captain Frederick Hellweg, United States Naval Observatory
1936	Levy Medal	Dr. Mayo D. Hersey, Massachusetts Institute of Technology

V.2
194

1936	Franklin Medal	Dr. Frank B. Jewett, New York City
1937	Henderson Medal	Dr. Rupen Eksergian, E. G. Budd Manufacturing Co.
1937	Longstreth Medal	Mr. Richard T. Erban, New York City
1937	Longstreth Medal	Mr. John S. Haug, Philadelphia, Pa.
1938	Levy Medal	Mr. Stewart S. Kurtz, Jr., Sun Oil Company
1938	Levy Medal	Mr. Alger L. Ward United Gas Improvement Co.
1939	Potts Medal	Dr. Newcomb K. Chaney, United Gas Improvement Co.
1939	Levy Medal	Prof. Kalman J. DeJuhasz, Pennsylvania State College
1941	Brown Medal	Dr. W. H. Carrier, Syracuse, New York
1941	Longstreth Medal	Mr. Benjamin J. Wilson, Oreland, Pa.

"We are very happy indeed to have you Medalists of former years with us again and we are looking to have you come back to us in succeeding years.

REPORT OF THE SECRETARY.

"I now have the pleasure and honor of presenting Dr. Henry Butler Allen, Secretary of the Institute and Director of its Museum, who will tell us something of the present-day activities of the Institute, the title of his talk being, 'How We Serve.'"

DR. HENRY BUTLER ALLEN: "Mr. President; Friends: 'How We Serve' can be quickly answered by the equally cryptic—'Well'; and one reason is we have such fine presidents. We have the Past President, Mr. Hayward, on my left, and Past President Dr. Staples on my right, and our present President, Charles Redding, presiding. And in addition this evening we have had our Hostess Committee to help us serve you.

"Once a reputation has been established for doing a good job, further orders come along. That is the case with our research work today, for the Army and Navy. Members of our Staff have shown what they could do; and now our only

difficulty lies in finding more workers. When it can be told, the members of The Franklin Institute will be proud of the direct part their Institute has had in winning this war. And of course we are proud of our boys who have gone into uniform.

"Our other activities have been 'stripped for action.' Our technical and patent libraries, under Walter Pertuch, are doing a flourishing business; our museum and planetarium, under the active supervision of Dick Lloyd, are furnishing amusement for thousands of children, old and young, and free relaxation for service men who come with their 'gals.' Our National Franklin Committee, with Miss Connelly on the lid here, is presenting Franklin's teachings to many more thousands.

"Two other signs which indicate we are well thought of are the continued financial contributions, under Jim Barnes, of our Friends of Franklin, and our Industrial Coöperation, with Bill Jackson at the helm.

"All of these efforts are going a long way to tide us over in these difficult times. Our stated meetings and those joint meetings with sister institutions have really been well attended considering the gas situation. Another very real indication of our usefulness in wartime, under the push of Mort Gibbons-Neff and Mrs. Harley, is the increase in our membership—1747 new members added since Medal Day a year ago—and not a few of them are here tonight. I wonder if those who joined us since Pearl Harbor would stand for a moment? Will those members who have joined within the past year please stand up? We want to get to know you well, and you to know your Institute. So come and see us often, for the better we serve, the happier we will be. Thank you."

PRESENTATION OF AWARDS

PRESIDENT REDDING: "Now we come to the very important part of tonight's program, the awards which have been made by The Franklin Institute to these scientists who are sitting up here at the head table.

"The first award is a Certificate of Merit to Carl S. Hornberger, who will be introduced by Mr. M. M. Price."

MR. PRICE: "Mr. President, the Board of Managers of The Franklin Institute, in accordance with the recommendations of its Committee on Science and the Arts, has voted to award a Certificate of Merit to Carl S. Hornberger, Central Scientific Company, Chicago, Illinois, 'In consideration of the excellence of design and construction of the Cenco Hypervac 100 Vacuum Pump, which has resulted in its wide recognition as a most useful piece of laboratory apparatus.'

"I present Mr. Carl S. Hornberger."

PRESIDENT REDDING: "Mr. Hornberger, by virtue of the power vested in me as President of The Franklin Institute of the State of Pennsylvania, I present to you this Certificate of Merit and the accompanying report.

"The next award is the Longstreth Medal which this year is awarded jointly to Robert Griffin De La Mater and William Schwemlein. These will be introduced by Charles D. Galloway."

MR. GALLOWAY: "Mr. President, I have the honor to present two gentlemen to you for the award of the Institute's Longstreth Medal, in consideration of the 'Design and successful application of a fluid brake which has made practical the drilling of wells to depths hitherto possible only with the greatest difficulty, and the accomplishment of this purpose with greater safety to life and property.' The brake which they designed and put into successful operation has relieved the mechanical braking of the enormous energy absorption required when controlling the lowering speed of well-casings weighing 200,000 pounds and upward at speeds from 10 to 20 feet per second to depths of 12,000 feet. The mechanical brake is still used to hold the load at rest and as an auxiliary to come to the full stop.

"I present Messrs. Robert Griffin De La Mater and William Schwemlein of the Parkersburg Rig and Reel Company, Parkersburg, West Virginia."

PRESIDENT REDDING: "Mr. De La Mater, by the same authority it is with much pleasure I present to you the Longstreth Medal, the accompanying certificate and the report of the Committee on Science and the Arts. I congratulate you, sir.

"And, Mr. Schwemlein, I have great pleasure presenting to you the Longstreth Medal, the accompanying certificate and the report of the Committee on Science and the Arts. I congratulate you, sir.

"The next award is the Wetherill Medal, to be presented to Robert Howland Leach, who will be introduced by W. B. Coleman."

MR. COLEMAN: "Mr. President, Members and Friends of the Institute: It is my privilege and pleasure to introduce to you the recipient of the John Price Wetherill Medal. It has been awarded upon recommendation of the Committee to a man who has spent the best part of his technical career working with silver and its alloys, and I think it is safe to say that the results of his labors are being used in practically every branch of the armed services today. Therefore, 'In consideration of his development and perfection of silver alloys applicable for the brazing of similar and dissimilar metals, and exhibiting in the molten state marked properties of fluidity and wetting qualities and in the solid state producing a joint exhibiting, in light of the low melting point, outstanding ductility and tensile strength,' Mr. Robert Howland Leach, Vice-President, Handy and Harman, Bridgeport, Connecticut, is here tonight, at our request. Mr. Leach."

PRESIDENT REDDING: "Mr. Leach, again by the same authority, I have great pleasure in presenting to you the Wetherill Medal, its accompanying certificate and the report of the Committee on Science and the Arts. I congratulate you, sir."

MR. ROBERT HOWLAND LEACH: "I would like to express my deep appreciation for this honor which The Franklin Institute has conferred upon me. I cannot but think of the many things my associates working with me have contributed, and I would like to say I am thinking of them at the time this honor is being presented to me. I thank you and they and the Committee on Science and the Arts for the conferring of this great honor which I hope in the future I shall continue to merit."

PRESIDENT REDDING: "The next award, the Brown Medal, is awarded posthumously to Dr. Albert Kahn. It is to be received by Mrs. Herbert L. Winston, Dr. Kahn's daughter, at

V.2
194

the request of Mrs. Kahn, who would have been here to accept it herself but illness kept her at home. I recognize Professor C. M. Gay."

PROFESSOR GAY: "Mr. Chairman, your Committee requests that the Brown Medal be bestowed, posthumously, upon a gentleman whose career was phenomenal, whose influence was international, whose works are his own memorial. When this gentleman took architecture as his profession, he became a pioneer in the practical and efficient housing of modern industry. His resourcefulness and scientific accuracy made possible the first reinforced concrete factory in America. In the first World War he was appointed Architect-in-Chief for the Signal Corps of the United States Army. In the present war he was responsible for the design of air bases for use in all types of climate all over the world. His work was always based on the three factors of speed, economy, and utility of construction.

"None of his countless contributions to building were ever patented, for, like our own Benjamin Franklin, he believed mankind should have the use of them, and they were placed primarily at the disposal of architects and engineers engaged in war construction.

"Unhappily, Dr. Albert Kahn died before he learned of this new honor awaiting him. I, Mr. President, request that the Brown Medal be awarded posthumously to Albert Kahn, of the firm of architects and engineers bearing his name, in Detroit, Michigan, 'In recognition of his outstanding achievements in the development of industrial architecture,' and that it be presented to Mrs. Herbert L. Winston, his daughter, who is here to receive it in his behalf."

PRESIDENT REDDING: "Mrs. Winston, it is very gracious of you to come here this evening to receive this Medal which is in honor of the work your father performed during his lifetime. We are all sorry your mother could not be here to accept this award herself, but I have great pleasure in presenting to you, to give to her, the medal, the certificate and the report of the Committee on Science and the Arts.

"The next award is the Institute's Henderson Medal, to Mr. Harry Miller Pflager who will be introduced by Dr. R. Eksergian."

DR. EKSERGIAN: "Mr. President, the Henderson Medal is awarded by the Institute this year to a distinguished railway engineer who has contributed basically to the design of the modern locomotive as well as for pioneer work in the design and development of very large steel castings for railway use. The award is made 'In consideration of his outstanding engineering achievements as a pioneer in the design and development of large integral steel castings, contributing in part to the use of high capacity modern power units and resulting in reduced operation costs.' Mr. Harry Miller Pflager is Senior Vice-President, General Steel Castings Corporation, Granite City, Illinois. Mr. Pflager."

PRESIDENT REDDING: "Mr. Pflager, by the same authority I have great pleasure in presenting to you the Henderson Medal, the accompanying certificate, and the report of the Committee on Science and the Arts."

MR. HARRY MILLER PFLAGER: "Mr. President, I cannot express my deep appreciation of the great honor Franklin Institute has conferred upon me, and my only hope is that my works will prove worthy of the honor."

PRESIDENT REDDING: "The next award is the Levy Medal, to be awarded to Mr. Anders Henrik Bull who will be introduced by Dr. C. B. Bazzoni."

DR. BAZZONI: "Mr. President, the award of the Levy Medal is made each year to the author of a paper of special merit which has appeared in the JOURNAL of The Franklin Institute in the course of the preceding calendar year. Our regulations require that this paper shall contain material original with the author and be of both a theoretical and experimental nature. This work must also refer to a subject of fundamental importance.

"The paper which the Committee has selected as most nearly meeting the terms of this award for the year 1942 appeared in the June issue of the JOURNAL and was entitled 'Soil Pressure Distribution Along Flexible Foundations.' The subject dealt with in this paper is of prime importance, especially to engineers engaged in the design of foundations for buildings and of subway structures. The author presents a novel approach to the problem, enabling the engineer to predict the earth pressures in a minimum of calculation and

with a maximum of certainty. In addition to the calculations which rest on a firm theoretical basis, the author presents also a mechanical approach involving mechanical models.

"The author of this paper is a man well known to civil engineers. He is Assistant Engineer, Board of Transportation, of the City of New York. It gives me great pleasure, Mr. President, to present to you, as the recipient of the Levy Medal for the year 1942, Mr. Anders Henrik Bull."

PRESIDENT REDDING: "Mr. Bull, by the same authority, I am very happy to present to you the Levy Medal, its accompanying certificate, and the report of the Committee on Science and the Arts.

"I have already referred to the fact that one of our awards this year was made to a South American. That award is the Potts Medal, to Senor Francisco Ballén. The award will be received by the Peruvian Consul, I now call on Mr. Lionel F. Levy."

MR. LEVY: "It is with great pleasure, Mr. President, that I bring to your attention an award to an outstanding scientist of Peru. Nature had provided, on islands off the coast of his country, great deposits of guano. It was known and used from the time of the Incas on, as a highly effective fertilizer. When its superior virtues became known to the outside world, the deposits were speedily mined in excess of replenishment, almost to exhaustion, and this valuable, and to Peru, very profitable industry, was practically ruined. The Peruvian Government wisely resorted to a program of restoration and protection. Some twenty-five years of scientific study by our Medalist of bird life, marine life, ocean and wind currents and other factors, bringing fish and other bird food to those waters in great quantity, and adoption of resulting measures recommended by him, including improved mining methods to eliminate undue interference with bird life and breeding, have resulted in making this very valuable fertilizer again available both to his own country and to the world at large.

"I have the pleasure, Mr. President, to present, in absentia, Senor Francisco Ballén, Director of the Guano Administration of Lima, Peru, to receive the Potts Medal, 'In consideration of his leadership in the scientific restoration of a valuable natural resource of high value to his country.'

"His own busy life and difficulties of travel prevented him from coming here today. However, we are honored to have his friend and countryman, the Honorable José Varela y Arias, Consul for Peru in Philadelphia, who will receive the Medal on behalf of Senor Ballén."

PRESIDENT REDDING: "Mr. Consul, we appreciate very much indeed your coming here this evening to join with us in these exercises, to receive on behalf of your countryman this Potts Medal, the certificate and the report of the Committee of Science and the Arts, which I will ask you to transmit to Senor Ballén at your convenience."

THE HON. JOSÉ VARELA (Consul for Peru in Philadelphia): "In accepting this honor conferred on my countryman, Senor Francisco Ballén, Director of the Guano Administration of Lima, Peru, I give my thanks to The Franklin Institute, to the President and all its supporters. Thank you."

PRESIDENT REDDING: "The next award is the Potts Medal to be presented to Dr. Paul Renno Heyl, who will be introduced by Dr. W. R. Wright."

DR. WRIGHT: "Mr. President, Dr. Heyl is a native of these parts. He is a graduate of the University of Pennsylvania. About twenty-five years ago he was the recipient of the Boyden Premium of The Franklin Institute, which award is offered in connection with problems concerning the velocity of light in stellar space.

"In about 1920 we loaned him to the Bureau of Standards, where for almost twenty years he has been carrying out experiments to a degree of accuracy that has seldom been equaled. He returns tonight as a recipient of the Potts Medal 'In recognition of his numerous contributions to physical science with particular reference to the extremely precise measurements which he has made in the field of gravitation.'"

PRESIDENT REDDING: "Dr. Heyl, I am very happy for the opportunity of presenting to you the Potts Medal, the accompanying certificate, and the report of the Committee on Science and the Arts."

DR. HEYL: "Thank you, Mr. President. An old Philadelphian always appreciates keenly an honor from The Franklin Institute."

V.2
194

PRESIDENT REDDING: "The next award is the Cresson Medal to Dr. Charles Metcalf Allen, who will be introduced by Mr. Joel D. Justin."

MR. JUSTIN: "Mr. President, the historic Cresson Medal has been awarded by The Franklin Institute since 1848. This year the man chosen to join the other distinguished recipents is the Director of the Alden Hydraulic Laboratory of Worcester Polytechnic Institute. He is internationally known as a hydraulic engineer, and particularly as the author of flow measurements. In and about his laboratory one finds many unique devices and schemes for saving time and trouble which are illustrative of his originality and initiative ability.

"Mr. President, 'In consideration of his origination of the salt velocity method for the measurement of the flow of water in conduits and for his contributions to the science of hydraulics,' I present Professor Charles Metcalf Allen, of the Worcester Polytechnic Institute."

PRESIDENT REDDING: "Dr. Allen, it is my great privilege to present to you the Elliott Cresson Gold Medal and certificate and a copy of the report of the Committee."

DR. ALLEN: "Thank you, Mr. President; and in order to have the record straight, I would like to say on behalf of a large number of assistants I have had in all these years and also on behalf of the Institute with which I have been in one way or another connected since 1891, I thank you."

PRESENTATION OF FRANKLIN MEDALS.

PRESIDENT REDDING: "The highest award which The Franklin Institute gives is the Franklin Medal, which is always accompanied by a Certificate of Honorary Membership.

"This year two such medals are being awarded, the first to Dr. George Washington Pierce, who will be introduced by Mr. Stuart Ballantine."

MR. BALLANTINE: "Mr. President, in view of the part which radio is playing in our daily lives and its important influence in the conduct of the present war, it seems particularly appropriate that the Institute has chosen, as one of the Franklin Medalists for this year, a distinguished pioneer in this field.

"Dr. George Washington Pierce has occupied the chairs of Rumford Professor of Physics and Gordon McKay Professor of Electrical Communication over a long period of years at Harvard, and many generations of students from all over the world have passed through his classes. His indirect influence on the art of radio communication through his students is fully as important as his own inventions and contributions. His courses in radio telegraphy and electric oscillations were the first to be given anywhere in this country and were the models upon which similar courses in other universities were later built.

"His technical contributions to this art have been many and varied, but time will permit reference to only a few: Early work on the rectifying properties of crystal detectors; mathematical calculation of the radiation properties of wireless antennæ; invention of a mercury-vapor tube which was the prototype of the thyratron later developed by A. W. Hull, and one of the most important electronic devices; invention of a method of recording sound on film by means of this tube, which was perhaps the first practical method of producing talking motion pictures; work on electric filters and electric compensators for underwater signalling and submarine detection: This work was done during the last World War as a member of a group of scientists working for the Government at New London. In this field of signalling by means of so-called supersonic sounds, having frequencies above the range of the human ear, Pierce has been preeminent. But perhaps his most important and widely used invention is the quartz piezo-electric oscillator. This device is employed in radio transmitters for accurately controlling the emitted frequency to a few parts in ten million. In this way broadcasting stations and other transmitters are kept rigorously in the groove so that they will not encroach upon the frequencies of other stations and cause interference. Millions of these oscillators are now in use, principally in military applications. Pierce also devised another way of doing the same thing by utilizing the magneto-striction effect in nickel and nichrome. This has important application in submarine signalling and submarine detection. He has published about fifty scientific papers, two textbooks on electric oscillations and has been granted over 50 patents.

"Therefore, 'In recognition of his outstanding inventions and contributions in the field of electric communication, and his inspiring influence as a great teacher' I take pleasure in presenting George Washington Pierce as a candidate for the Franklin Medal."

PRESIDENT REDDING: "Dr. Pierce, by virtue of the power vested in me as President of The Franklin Institute of the State of Pennsylvania, I have the honor to confer upon you the Franklin Medal with its accompanying certificate and report, and also Honorary Membership in The Franklin Institute.

"Dr. Pierce will now address us on the 'Songs of Insects.'[1]

PRESIDENT REDDING: "Thank you, very much. We certainly enjoyed your talk.

"The final award, another Franklin Medal and Certificate of Honorary Membership, is to be given to Dr. Harold Clayton Urey, who will be introduced by Dr. Frederick Palmer."

DR. PALMER: "Mr. President, I have the honor to present, as a candidate for the Franklin Medal, Harold Clayton Urey, Professor of Chemistry and Executive Officer, Department of Chemistry, Columbia University.

"In 1931 it had been suggested that normal hydrogen might be a mixture of two different kinds of hydrogen, one the ordinary light variety, the other a heavy variety each atom of which had twice the mass of one of the light atoms. If this were true, it was estimated that there was only one heavy atom to 4500 light atoms in normal hydrogen. At that time there was no known means of detecting the presence of atoms with such a small relative abundance as this. It was mere speculation.

"In attempting to establish the actual existence of atoms of heavy hydrogen Doctor Urey attacked the problem from two directions: (1) he devised a method of concentrating the heavy atoms in a sample of hydrogen so as to increase their relative abundance; (2) he used a method of detecting their presence in such a sample that was far more sensitive than any method previously employed.

"Thermodynamic theory led him to suspect that light hydrogen should have a somewhat lower boiling point than

[1] Dr. Pierce's paper, "Songs of Insects," appears on page 141 of this issue.

heavy hydrogen; hence the light atoms should boil away more rapidly than the heavy ones leaving the residue enriched with heavy atoms. This process, which is called fractional distillation, he used very effectively.

"Bohr's spectral theory led Doctor Urey to predict that the spectrum of heavy hydrogen should be just like that of light hydrogen except that each spectral line should be of slightly shorter wave-length. By the use of a large spectrograph at the Bureau of Standards Doctor Urey obtained photographs of the spectrum from an enriched sample of hydrogen. These photographs showed each line in the spectrum of light hydrogen accompanied on the shorter side by a companion line, and the measured distance between them agreed remarkably with that which he had calculated from theory. Thus the existence of atoms of heavy hydrogen was established.

"The physical, chemical, and biological behavior of the heavy atoms proved to be, in certain respects, different from that of the light atoms; thus Doctor Urey's achievement has opened up vast new fields of research in these sciences.

"Hence 'In recognition of his discovery of an isotope of hydrogen of mass 2 which has resulted in the opening of new fields of knowledge in three of the physical sciences' I take pleasure in presenting Harold Clayton Urey, discoverer of heavy hydrogen, now known as Deuterium."

PRESIDENT REDDING: "Dr. Urey, by virtue of the power vested in me as President of The Franklin Institute of the State of Pennsylvania, I have the honor to confer upon you the Franklin Medal, with the certificate and copy of the report, and also Honorary Membership in The Franklin Institute.

"Dr. Urey will now present his paper, 'The First Ten Years of Heavy Hydrogen': Dr. Urey."[2]

President Redding, after expressing appreciation to Dr. Urey, then called upon Mrs. Braun to conclude the meeting with the playing of 'America.' Before this was done, however, he thanked members and friends for their attendance and expressed the hope that next year would bring them back again, together with the Medalists who had received their awards, and thus participated for the first time, this year.

[2] Dr. Urey's paper, "The First Ten Years of Heavy Hydrogen," will appear in a forthcoming issue of the JOURNAL OF THE FRANKLIN INSTITUTE.

LIBRARY NOTES.

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat prints of any material in the collections can be supplied on request. The average cost for a print 9 × 14 inches is thirty-five cents.

The library and the reading room are open on Mondays, Tuesdays, Wednesdays and Fridays from nine o'clock A.M. until five o'clock P.M., Thursdays from two until ten o'clock P.M. and Saturdays from nine until twelve o'clock noon.

RECENT ADDITIONS.

AERONAUTICS.

REDPATH, PETER H., AND JAMES M. COBURN. Air Transport Navigation. 1943.

ARCHITECTURE AND BUILDING.

American Concrete Institute. Proceedings. Volume 38. 1942.

FRYKLUND, VERNE C., AND CHARLES H. SECHREST. Materials of Construction. 1943.

PLUM, SVEND. Plumbing Practice and Design. Volume 2. 1943.

CHEMISTRY AND CHEMICAL TECHNOLOGY.

American Association of Textile Chemists and Colorists. Year Book. Volume 19. 1942.

BARRON, HARRY. Modern Synthetic Rubbers. 1942.

BURK, R. E., AND OLIVER GRUMMITT, Editors. The Chemistry of Large Molecules. 1943.

MACY, RUDOLPH. Organic Chemistry Simplified. 1943.

MORTON, AVERY ADRIAN. Laboratory Technique in Organic Chemistry. 1938.

SMITH, PAUL I. Synthetic Adhesives. 1943.

SWIETOSLAWSKI, W. Coke Formation Process and Physico-Chemical Properties of Coals. 1942.

THEROUX, FRANK R., EDWARD F. ELDRIDGE, AND W. LE ROY MALLMANN. Laboratory Manual for Chemical and Bacterial Analysis of Water and Sewage. Third Edition. 1943.

TURRENTINE, J. W. Potash in North America. 1943.

WRIGHT, R. H. Manual of Laboratory Glass-Blowing. 1943.

ENCYCLOPEDIAS.

Britannica Book of the Year. 1943.

ELECTRIC ENGINEERING.

RIDER, JOHN F. Perpetual Trouble Shooter's Manual. Volume 13. 1942.

STURLEY, K. R. Radio Receiver Design. Part 1. 1943.

ENGINEERING.

Engineering Index 1942. 1943.

MANUFACTURES.

THISELL, A. GIDEON. Science of Watch Repairing Simplified. No date.

MARINE ENGINEERING AND NAVIGATION.

KELLS, LYMAN M., WILLIS F. KERN, AND JAMES R. BLAND. Piloting and maneuvering of Ships. 1943.

LAMB, JOHN. The Running and Maintenance of the Marine Diesel Engine. 1942.

MATHEMATICS.

JAHNKE, EUGENE, AND FRITZ EMDE. Tables of Functions with Formulae and Curves. 1943.

WEBSTER, ARTHUR GORDON. Partial Differential Equations of Mathematical Physics. Second Corrected Edition. 1933.

WILKS, S. S. Mathematical Statistics. 1943.

MECHANICAL ENGINEERING.

American Society of Refrigerating Engineers. Refrigerating Data Book. 1943.

CLARK, WILL M. A Manual of Mechanical Movements. 1943.

METALLURGY.

BAËZA, WALTER J. A Course in Powder Metallurgy. 1943.

WILKINS, R. A., AND E. S. BUNN. Copper and Copper Base Alloys. First Edition. 1943.

METEOROLOGY.

BARBER, CHARLES WILLIAM. An Illustrated Outline of Weather Science. 1943.

MILITARY SCIENCE.

VAN VALKENBURG, SAMUEL, Editor. America at War: a Geographical Analysis. 1943.

PHOTOGRAPHY.

U. S. Camera. 1943.

PHYSICS.

BOELTER, L. M. K., V. H. CHERRY AND H. A. JOHNSON. Heat Transfer. 1942.

BRUNAUER, STEPHEN. The Adsorption of Gases and Vapors. Volume I. 1943.

KNAPP, E. J. Basic Physics for Pilots and Flight Crews. 1943.

OLSON, HARRY F. Dynamical Analogies. 1943.

TWYMAN, F. Prism and Lens Making. No date.

TRANSPORTATION.

ST. CLAIR, LABERT. Since Time Began. Transportation. New and Revised Edition. 1942.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

The Determination of Purines in Protein Material.—ALFRED BLOCH. The determination of the total purine content of protein material requires the liberation of the bound purines by acid hydrolysis, the separation and purification of the purines, and finally the estimation of their nitrogen content.

Graff and Maculla (*Journal of Biological Chemistry*, **110**: 71, 1935) recommend hydrolysis of the protein and liberation of the purines by means of hydrochloric-formic acid mixture, and precipitation of the purines by means of cuprous oxide suspension from a citrate buffered solution at pH 5. In order to purify the precipitate from adsorbed non-purine compounds, they then dissolve it in trichloroacetic acid solution and after destruction of the trichloroacetic acid, reprecipitate the copper-purine compounds under conditions similar to those for the first precipitation.

In applying this method to mixtures of casein and purines,* quite low recoveries were obtained, although they were satisfactory when the protein was omitted and the use of trichloroacetic acid thereby avoided. However, it was found that the initial copper-purine precipitate could be redissolved in the same hydrochloric-formic acid mixture originally used to hydrolyze the protein, the solution neutralized, buffered, and otherwise treated in the same way as in the first stage of the analysis, and the purines precipitated for a second time. The reprecipitated copper-purine compounds were washed

* The following purine products were used:

Guanine hydrochloride	$C_5H_5N_5O.HCl.H_2O$	Eastman Kodak Co.
Adenine sulfate	$(C_5H_5N_5)_2.H_2SO_4.2H_2O$	Eastman Kodak Co.
Xanthine	$C_5H_4N_4O_2$	The Mearl Corp.
Hypoxanthine	$C_5H_4N_4O$	Hoffman-LaRoche
Uric acid	$C_5H_4N_4O_3$	Pfanstiehl Chem. Co.

and nitrogen determined on the residue by the Kjeldahl method. Recoveries were then satisfactory.

It was also found advantageous to substitute alkacid test paper (Fisher Scientific Co.) for the Congo Red used as internal indication by Graff and Maculla in adjusting the pH before precipitation of the copper-purines. Alkali was added to the originally acid solution until the test paper (spot test) turned to a slight yellow-green, which, after addition of the required citrate buffer, corresponds to a pH of 5–6. Owing to masking of the Congo Red color by melanine or other colored material, the adjustment of pH by means of the internal indicator did not give satisfactory results, as shown by glass-electrode tests.

Representative recoveries of purines added to casein, obtained by this modification of the Graff-Maculla method are shown in Table I. In general four analyses were carried out simultaneously.

TABLE I.

Purine N Values Obtained.

Material, g.	Purine Added	Purine N added, mg.	Purine N found, mg.	Recovery, %
Casein 1	Adenine	0.946	0.927	95.8
Casein 0.5	Adenine	0.506	0.497	98.2
Casein 0.5	Adenine	1.012	1.066	105.3
Casein 1	Guanine	0.660	0.650	98.5
Casein 0.5	Guanine	0.630	0.614	97.5
Casein 0.5	Guanine	0.315	0.304	96.5
Casein 0.5	Guanine	0.158	0.154	97.5
Casein 1	Xanthine	0.828	0.818	98.8
Casein 1	Hypoxanthine	1.104	1.118	101.3
Casein 1	Hypoxanthine	0.552	0.520	94.2
Casein 1	Hypoxanthine	0.276	0.284	102.9
Casein 0.5	Mixture of	0.312	0.297	95.2
Casein 0.5	Hypoxanthine,	0.312	0.308	98.7
Casein 0.5	Xanthine,	0.312	0.321	102.9
Casein 0.5	Guanine and	0.208	0.217	104.3
Casein 0.5	Adenine	0.104	0.109	104.8

Recovery of Uric Acid.—Though not mentioned in the paper of Graff and Maculla, uric acid is quantitatively precipitated with copper sulfate and bisulfite, according to Krueger and Schmidt (*Hoppe-Seyler's Zeitschrift fuer physiologische Chemie*, **45**: 1, 1905). The recovery of uric acid by

TABLE II.

Recovery of Uric Acid.

Uric acid N added, mg.	N found in the 1st Cu_2O-ppt., mg.	N found in the 2nd Cu_2O-ppt., mg.	Recovery, %
with 900 mg. casein			
0.245	0.171	—	69.8
0.408	0.328	—	80.4
0.245	—	0.093	38.0
0.408	—	0.136	33.3
0.706	—	0.186	26.4

the modified procedure used in this laboratory was inconsistent and incomplete (see Table II) showing that a false value may be obtained for purine nitrogen if this compound is present in substantial amounts. The duration of the hydrolysis with HCl.HCOOH does not influence the recovery which was always about 2/3 when hydrolyzed for 0, 30, 60, 120 and 180 minutes (one precipitation with cuprous oxide).

The Detection of Minute Amounts of Adenine and Guanine in Casein.—ALFRED BLOCH. Casein was used by Graff and Maculla (*Journal of Biological Chemistry*, **110**: 71, 1935), by Edlbacher and Jucker (*Hoppe-Seyler's Zeitschrift fuer physiologische Chemie*, **240**: 78, 1936) and by us (Bloch, Alfred, *The Determination of Purines in Protein Material*, *Journal of the Franklin Institute*, **235**: 217, August, 1943) as the purine-free standard protein-material to which known amounts of purines were added for the determination of purine recovery. The casein employed by others is not described; we used the "casein according to Hammarsten" (Eimer and Amend). No hint that pure casein contains purines could be found in the literature. Analyses of "commercial casein" have been reported by Crandall and Young (*Biochemical Journal*, **32**: 1133, 1938) who found 1 mg. purine nitrogen per 100 g. casein when determined by the procedure of Krueger and Schmidt (*Hoppe-Seyler's Zeitschrift fuer physiologische Chemie*, **45**: 1, 1905) and 14 mg. per 100 g. when using the method of Graff and Maculla.

"Purine"-determinations with 1 g. samples of our casein gave persistent values for nitrogen slightly higher than blanks

with the reagents alone: 0.02 to 0.04 mg. N (with one sample 0.08 mg.), corresponding to 2 to 4 mg. "purine" nitrogen per 100 g. casein. A sample marked "casein, highest purity, Pfanstiehl" gave 2.94 mg. purine N per 100 g. sample; "casein Difco" 6.85 mg., "S.M.A.C." gave 4.18 mg. purine N per 100 g. sample. Edlbacher and Jucker expressed the opinion that albumoses, formed during hydrolysis of casein might come down with the copper-purine precipitate of the added purine. To ascertain whether or not the nitrogen found represents purine nitrogen, the material precipitated with the cuprous oxide from two samples was isolated and was identified as a mixture of adenine and guanine. The original source of the adenine and guanine is unknown. Conceivably, they could be constituents of the casein molecule, but in the absence of supporting evidence, it is preferable to consider them as impurities in the casein samples, the amounts varying as indicated with different casein samples. Since the preparation and purification of casein from milk are accomplished principally by alternate precipitation and resolution, it is most likely that the presence of the purines is due to their high adsorbability to protein. In fact, a minute amount may be retained by the protein even in the most thoroughly purified casein.

Isolation of Adenine and Guanine.—Two lots, 60 g. of casein according to Hammarsten (Eimer and Amend) and 200 g. of casein (Baker, technical grade), were separately treated. The cuprous oxide-purine complex from the second precipitation in the separation of purines was dissolved in diluted HCl. The solution was negative to both the Sullivan and the sulfur tests and this showed that no cysteine was present. It was also negative to the biuret reaction. The copper was removed with H_2S and the excess H_2S was boiled off. Kjeldahl determinations on aliquot parts of the clear solutions gave 2.8 mg. N per 100 g. casein on sample marked "according to Hammarsten" and 3.4 mg. N per 100 g. casein on sample marked "Baker, technical grade."

The hydrochloric acid solution was made slightly alkaline with ammonia (methyl red), concentrated, and after addition of more ammonia a coarse crystalline precipitate formed. The precipitate (A) was separated from the mother liquor (B).

The precipitate (A) dissolved readily in 2/3 N HCl and recrystallized upon addition of excess ammonia. The crystals did not melt nor decompose below 320°, the highest temperature used. They showed the same structure of plates and needles and the same interference colors under the polarization microscope as did pure guanine, the only purine base precipitated by ammonia. The samples gave the same positive reaction as guanine to the murexide test (according to Rosenthaler, L., *Der Nachweis organischer Verbindungen*, Stuttgart, p. 568, 1923). The presence of xanthine, which would give the same reaction, is excluded since it is very easily soluble in alkalies. The solution (A) in HCl, upon addition of sodium picrate, gave dark yellow, coarse, silky needles in brush formation, such as are typical for guanine picrate. Not enough material was obtained for elementary analysis after recrystallization. For comparison, guanine picrate was prepared from guanine hydrochloride (Eastman). The product, recrystallized 4 times from 25 per cent. acetic acid and washed and dried at 110°, had the same structure and appearance as the picrates from casein. Guanine picrate, $C_6H_5N_5O.C_6H_3N_3O_7$, 29.47 per cent. N theoretical, gave 29.40 per cent. N on analysis by Kjeldahl's method.

According to Meyer-Jacobson (*Lehrbuch der organischen Chemie*, Berlin, 2^{III}, p. 1334, 1920), guanine picrate decomposes at 255–260°, but Rosenthaler records 190°. When heating at the rate of 5° or of 20° a minute a slight sintering of the surface touching the capillary wall was observed at 240–242° (all figures uncorrected), above 250° gradual and above 280° faster darkening, with no further change between 296° and 320°. On the Kofler hot stage at about 200° a fine film of sublimate was visible, which melted at 240–245°. The main bulk of crystals lost their luster and darkened gradually with no further change up to 330°. The sublimate represented only a negligible fraction of the material, which fraction may represent traces of adenine sticking to the guanine as the properties of adenine picrate described below suggest.

The same decomposition process was found in the capillary and on the hot stage for the picrates obtained from pure guanine and from the guanine isolated from the two lots of

casein. Mixed decomposition points (because of lack of material) could be run only with the picrate from purines isolated from casein marked "Technical."

For further identification, ultraviolet absorption spectra were taken of known guanine picrates and of that from casein ("Technical"). The curves obtained on 0.02 per cent. picrate solutions in 3 per cent. acetic acid agree very closely. The slight variation in extinction coefficient values is due to differences in concentration, as only 0.347 mg. of the unknown picrate was dissolved (see Fig. 1).

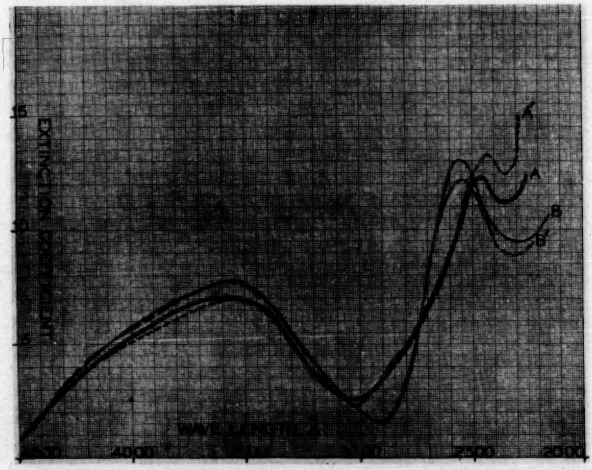


FIG. 1. Ultraviolet absorption curves of guanine picrate (0.02 per cent. solution in 3 per cent. acetic acid) and adenine picrate (0.019 per cent. aqueous solution).

A Known guanine picrate.
A′ Unknown guanine.
B Known adenine picrate.
B′ Unknown adenine picrate.

The concentrated mother liquor (B) gave very little precipitate with sodium picrate because of the presence of much ammonium chloride. The solution, freed from picric

acid, was treated with ammoniacal silver nitrate and the precipitate extracted with 0.5 N HCl. To the hydrochloric acid extract, after evaporation to dryness and solution in water, was added sodium picrate; at once fine, yellow, hair-like needles with the typical structure of adenine picrate crystallized from the solution. After 10 minutes they were separated and recrystallized from diluted acetic acid and finally from water.

For comparison, adenine picrate was prepared from adenine sulfate (Eastman), recrystallized four times from water and dried. Adenine picrate, $C_5H_5N_5 . C_6H_3N_3O_7$, 30.77 per cent. N theoretical, gave 30.78 per cent. N by Kjeldahl's method. Not enough material from the unknown picrates was obtained to run elementary analyses.

The decomposition points of the known and suspected adenine picrates were as follows: known 287°; unknown 282–4°; mixture of known and unknown 285–290°. The evolution of gas, as mentioned by Vickery and Leavenworth (*Journal of Biological Chemistry*, **63**: 579, 1925), occurred at the following temperatures: known 296°; unknown 294°; mixture of known and unknown 295°. These melting points were carried out in capillaries and at a rate of 5 or 20 degrees per minute.

When heating on the Kofler hot stage at the rate of 2° to 3° a minute the adenine picrate, like adenine, sublimed. At about 180° a fine yellow film was visible on the cover glass and at 220–230° all but traces of the substance had sublimed to needles, which melted at 248° and darkened gradually with no other change until 320° was reached. After cooling, the sublimate, consisting of fine needles, again melted at 246°. The sublimed picrate, 30.36 per cent. N by Kjeldahl's procedure, decomposed in the capillary at a slightly higher temperature: sublimate from known 291°; mixture of known and derived sublimate 287–291°; evolution of gas-sublimate from known 298°; evolution of gas-mixture of known and derived sublimate 299°. The same rate of sublimation was found for the unknown adenine picrates.

The ultraviolet absorption spectra of the known and the unknown adenine picrate from casein (Technical) in 0.019 per cent. aqueous solutions coincide very closely (see Fig. 1).

The mother liquor of the adenine picrate, absolutely clear after standing for 24 hours, upon addition of nitric acid and silver nitrate showed a slight flocculation which was probably the silver picrate of xanthine or hypoxanthine; the small quantity allowed no identification. The isolation of the individual purines from these small amounts gave us no quantitative yields, the adenine obviously prevailing.

BOOK REVIEWS.

POTASH IN NORTH AMERICA, by J. W. Turrentine, Ph.D. 186 pages, illustrations, 16 × 23 cms. New York, Reinhold Publishing Corp., 1943. Price $3.50.

The title of this book coming at this time is a reminder of conditions in this country 25 years ago during World War I. At that time we found ourselves without this very necessary substance to conduct operations for the advancement of the war. The situation was somewhat similar to the rubber situation today and parallels have been drawn to the effect that the rubber industry will have the future that the potash industry held.

This book is a continuation of the story published in 1926 in a book of the same title. That story covered the years from 1911 when legislation was enacted by the Federal Congress inaugurating potash researches, and records the successful efforts of 1914–18 and the subsequent development of the industry. The book at hand takes up the story where it was dropped in 1926 and relates what has happened to date. It tells of the exploration for potash deposits, examination of saline stratification, the estimated reserves, current operations, and foreign developments. Reference is made to the American Potash Institute, its purposes and work. A review is made of the uses of potash, and the technology of potash production is explained as a part of a detailed account of the several corporate units comprising the American potash industry. These cover the operations of the American Potash and Chemical Corporation in the refining of Searles Lake brine, the Trona process, the mining and refining of Sylvinite, of Langbeinite, and the refining of Salduro Marsh brine.

The story is well written and adequately presents the development and status of the industry. It is an American Chemical Society monograph.

R. H. OPPERMANN.

BASIC PHYSICS FOR PILOTS AND FLIGHT CREWS, by E. J. Knapp, Ph.D. 118 pages, charts and diagrams, 13 × 19 cms. New York, Prentice-Hall, Inc., 1943. Price $1.25.

Now, probably more than ever before, is it necessary in many occupations to quickly obtain the most essential points of various scientific topics. To assemble these points is a very difficult task, as is their proper presentation to facilitate understanding. Both processes, to produce results, require a great deal of experience as a background. This little book, as its title implies, gives the essential points of physics for application in preparation for meteorology, theory of flight, and engine operation. The author states it is designed to meet the requirements of primary flight training or air cadet training.

A brief treatment on atmospheric phenomena is first introduced, then brief treatments on units of measure, vectors and accelerated motion are given. Fluids at rest and work, energy, power and friction are subjects which complete part one of the book. Part two has to do with fluids in motion, heat, gases, change of state and atmospheric humidity. In the back there is a subject index. The

work is admirably compiled, well presented, and this fact together with the carefully selected exercises and problems, to which answers are supplied, make a compact and informative course in the subject.

R. H. OPPERMANN.

A COURSE IN POWDER METALLURGY, by Walter J. Baeza. 212 pages, illustrations, 16 × 24 cms. New York, Reinhold Publishing Corp., 1943. Price $3.50.

The Delhi Pillar in India which has been the subject of much interest points to a probable early application of powder metallurgy. Today very astonishing results are being obtained in many fields of activity. The study of powder metallurgy has much promise, especially when much of the unknown is brought to light. This book presents information on the subject which may be used as a starting point for research in as well as application of powder metallurgy.

An idea of the importance and present status of the subject may be obtained by the classifications of the general applications given in this book. Powder metallurgy may be used to produce solid ingots from highly refractory metals which cannot be commercially fused in available furnace equipment. It may be used to combine in a single unit metals of greatly disparate melting points or highly distinctive characteristics with the object of producing a unit with the characteristics of the several combined metals unmodified. It may be used to make precise-dimension products which are so hard or brittle that conventional shaping operations are difficult and sometimes impossible. It may be used for the production of units having characteristics unobtainable by conventional methods, such as heavy metal, porous metals, and electronic products. It may be used for laminated parts with a more intimate binding than would be possible by conventional methods of attaching one metal to another. And it may supplant machined precision units, when cheaper or more accurate than machining, or when there is a shortage of lathes or skilled machinists.

The book discusses methods for the production of metal powders, their specification, classification, and cohesion. Manufacturing problems are given attention as well. For the purpose of assisting the progress of the industry there is indicated what a college course in powder metallurgy should contain, its objectives, student qualifications, college equipment, and instruction. The work also examines costs for laboratory training and general instructions to students. And last there are laid out descriptions for fifteen experiments.

The book should do much in furthering the subject of powder metallurgy and anyone interested should by all means give it close serutiny.

R. H. OPPERMANN.

DICTIONARY OF SCIENCE AND TECHNOLOGY, by Maxim Newmark, Ph.D. 386 pages, 16 × 23 cms. New York, Philosophical Library, Inc., 1943. Price $6.00.

Every translator knows well the need of reference works and dictionaries of technical terms and phrases. A few of these are available which are more or less standard, but there is still a need due partly to the rapid rate of obsolescence of such works and the new sciences and branches of science making their appearance. The author of this work states that it is the outgrowth of some ten years-

experience in teaching the reading and translation of scientific and technical material in foreign languages to the students of a polytechnical institution. Beginning as a translator's vocabulary it grew vertically in the number of terms and horizontally in the coverage of subjects until in its present form of this book it covers such subjects as plane and solid geometry, trigonometry and advanced algebra, materials and processes of industry, chemistry, physics, machine shop terms, mechanical drawing, etc. It has French, German, and Spanish equivalents, synonyms, and definitions of a basic list of some 10,000 English terms currently used in the polytechnical and scientific fields. It contains in all these languages approximately 45,000 terms. Each entry is numbered, and separate indexes of French, German and Spanish terms, correspondingly numbered, permit two way use of any of these languages with the English. Entries are listed in strict alphabetical order, the sole exception being compound terms, many of which are entered under parent terms to afford convenient reference of related subject matter. A feature to eliminate useless searching is a comprehensive system of cross-references offering a guide to the initial component of compound terms, to synonymous terms, and to colloquialisms.

Every reader and translator of these languages would do well to examine this work.

R. H. OPPERMANN.

MAGNETIC CIRCUITS AND TRANSFORMERS, by Members of the Staff of the Massachusetts Institute of Technology. 718 pages, charts and diagrams, 16 × 23 cms. New York, John Wiley & Sons, Inc., 1943. Price $6.50.

The subject of this book has often been termed by electrical engineers the most interesting in the entire field. Certainly it is an important fundamental; for magnetic circuits form the basis of electrical engineering and the transformer is an indispensable apparatus as well as one having the highest efficiency. These features of the subject make it one that must be presented to students with the utmost care, and it is with this in mind that the Members of the Staff of the Department of Electrical Engineering of Massachusetts Institute of Technology have cooperated and produced this text. It is the second volume in the Principles of Electrical Engineering series of books.

The treatment assumes that the reader has knowledge of electric-circuit theory. Mathematics through differential equations is freely used with stress laid on the means of describing quantitatively the consequences of physical phenomena rather than its use as a substitute for the explanation of physical phenomena—a valuable point in a text. There are two parts to the text as the title implies. Under the heading of magnetic circuits, the treatment opens with a discussion of the properties of the ferromagnetic materials of chief interest to the electrical engineer. Numerous quantitative data are given which form the basis for the solution of magnetic circuit problems treated later. Following this, as an aid to a better visualization of the phenomena involved, a theory of magnetism is described. Principles of magnetic-circuit computations are laid down and included with them is the subject matter of the special ingenious techniques for the solution of problems. There are also covered in order, losses in magnetic cores containing time-varying fluxes, alternating current excitation characteristics of iron-core reactors and transformers, model theory and design of iron-core

reactors, and thermal properties and heat flow. While at first glance the latter subject may seem a bit out of place, its coverage at this point is certainly justified as heating is rightly considered as a very important element in design. Reference is made in it to transient heating.

Part II treats on transformers beginning with elementary theory and general principles including cost considerations. The procedure then follows the order of physical features, leakage, theory of operation including voltage regulation and efficiency, autotransformers, economy in power system applications, self and mutual inductances, and coupled-circuit equations, frequency characteristics, instrument and tap-changing transformers, applications in telephone systems, three-phase connections and uses, harmonic phenomena, parallel operation, etc. A vision of what all these topics and more mean will give the correct impression that the subject of transformers is covered quite completely.

The many problems given for exercise are practical and interesting. As a matter of fact they put forth a challenge to the graduate engineer reading them. The presentation is clear and helpful. The subject index in the back adds much to the value of the book as a reference.

R. H. OPPERMANN.

APPLIED ELECTRONICS, by Members of the Staff of the Massachusetts Institute of Technology. 772 pages, charts and diagrams, 15 × 23 cms. New York, John Wiley and Sons, Inc., 1943. Price $6.50.

Electronics is a magic word today in industry. It is reaching into every branch of electrical engineering to an increasing extent and through this science into an ever widening variety of industries with astonishing results. Shortened processes are producing better materials in only a fraction of the former time. From an electrical engineering educational viewpoint it is difficult to state a more important subject today than electronics. The book at hand has for its purpose the enlargement of the fundamental principles of electrical engineering in the direction of the application of electronics. It is one of a series of books resulting from educational research by the Massachusetts Institute of Technology.

The subject is taken up by first examining the physical phenomena involved in electronic conduction. Under the topic entitled "Electron Ballistics" the dominant concept presented is the relative ease and precision with which the motion of a charged particle in free space may be controlled. This leads to the source of supply of electrons and the study of the behavior of the free electrons in a metal and the methods by which they may be liberated. Sufficient of the fundamental concepts is given of this to be helpful in understanding the operation of electronic devices. Likewise the bases for the understanding of electrical conduction through vacuum, gases and vapors are given involving instances in which most of the electrons and ions that participate in the discharge are supplied by the discharge itself. An exception to this is the Townsend discharge which opens the discussion on gaseous discharges.

What may be taken as the next large division of the book discusses the ways in which the physical phenomena involved in electronic conduction combine to determine the characteristics and limitations of some of the more important electron tubes and the resulting characteristics. This division is devoted first

to high-vacuum tubes and second to gas tubes. The characteristics described are later used in the analysis of the over-all behavior of electron tube circuits in which the tube is associated with other circuit elements to form a working system. The circuits mentioned are, in order, single-phase rectifier circuits, polyphase rectifiers, Class A single stage amplifiers, Cascade amplifiers, amplifiers of classes AB, B, and C vacuum tube oscillators. The last chapter of the book is given to the subject of modulation and demodulation or detection.

The book contains appendices with useful working information, a bibliography of bibliographies classified according to subjects, and an author and subject index. The work is well presented in construction and language. Simplicity prevails. The course represented by the book is not a short one but not so long as to preclude its practical value. Many problems at the end of chapters indicate that the student is expected to work hard with the result of having a thorough knowledge for the application of electronics.

R. H. OPPERMANN.

PUBLICATIONS RECEIVED.

Electronic Control of Resistance Welding, by George M. Chute. 389 pages, illustrations and diagrams, 16 × 23 cms. New York, The McGraw-Hill Book Company, 1943. Price $4.00.

Practical Physics, by Marsh W. White and Others. 365 pages, charts and diagrams, 16 × 23 cms. New York, McGraw-Hill Book Company, 1943. Price $2.50.

Handbook of Elementary Physics, by Robert Bruce Lindsay. 382 pages, charts and diagrams, 14 × 21 cms. New York, The Dryden Press, Inc., 1943. Price $2.25.

CURRENT TOPICS.

Underwater Resistance Welding of Stranded Wire.—(*Iron Age*, Vol. 51, No. 16.) At the East Pittsburgh works of the Westinghouse Elect. & Mfg. Co. underwater resistance welding has replaced many of the routine processes of soldering or brazing of stranded copper cable to terminals. This modification of a standard welding process has saved considerably in production costs by raising the output and improving the quality of finished products. Essentially, the process consists of welding by means of special tips while playing a stream of water on the stranded wire near the electrode. Water is applied in a volume sufficient to cool the work and prevent oxidation at the weld. Spot welding machines used for this work are usually designed with a special jig fastened around the stationary electrode to hold the parts firmly in their relative welding positions. Electrode tips as now used are made of carbon, molybdenum and cupaloy. Increased tip life is obtained by mounting them in water cooled holders. For welds that do not require flux, cupaloy tips with molybdenum inserts have been found most satisfactory. Carbon tips are required where flux is used. Inserts that contact the wire are grooved to prevent flattening the cable. By means of a foot pedal, the operator initiates the lowering of the upper electrode to complete the weld. The machine is air operated and is timed by electronic controls which insure a quality weld. The completed assembly may be removed immediately without the use of pliers or gloves. A noticeable comparison in completed assemblies by the old and new methods is that the wire cables remain flexible to the point of weld in the under water resistance weld. In either the soldering or the brazing process molten metal is drawn into the hot cable by capillary action and diminishes the flexibility of the cable. Cooling the parts with water not only makes handling easier but heat is removed from immediate area of the weld rather than passing to more distant areas. Thus the original properties of the individual parts are not changed.

R. H. O.

" Fish-Eye " Camera Provides Quick Solution of Lighting Problems.—Photographs made with a " fish-eye " camera which show everything in a room above its level, enable engineers to solve quickly problems of illumination which would otherwise require elaborate and lengthy calculations. Speaking recently before a

joint meeting of the Rochester Section of the Optical Society of America and the Western New York Section of the Illuminating Engineering Society, Frank Benford, General Electric Research Laboratory physicist who developed the new camera described its uses. He said that this "illuminagraphic camera" will be especially valuable for studying the illumination in airplane and other factories built without windows and lighted with large areas of fluorescent lamps. In an experimental model, Mr. Benford uses a camera of customary type, mounted on an iron stand, and pointed toward a curved mirror which is high in the center. The curve of this mirror had to be adjusted very accurately. The result is that a photograph taken of its reflection shows the area of extended light sources such as a window which shows the sky, or a bank of fluorescent lamps, in exact proportion to the amount of light which a surface at the location of the mirror receives from the light source. Even if the area from which the illumination comes is irregular it can be measured easily on the photographs with a planimeter. When the brightness of the source is known, the actual illumination at the mirror can be determined. Both the camera itself and the support are visible in the picture, but they make up a small area, and ordinarily would not affect the result. It is not necessary that the mirror be horizontal. It can, for example, be placed at an angle, as to measure the illumination of a sloping drawing board. By determining the amount of light received from a window, and between its various parts, such as the upper and lower sashes, the most efficient lighting can be planned.

R. H. O.

Spotlight Turned on Mica Shortage by New Development.—Recent announcement of the development of "Polectron" Synthetic by the Americanized General Aniline and Film Corp., as the basic substance in mica replacement materials, has sharply focused attention on the important place strategic mica, largely imported from India, is playing in the war. Shortwave broadcasting depends on mica. Field communication and detection devices, in fact the whole field of radio, radar and electronic equipment, depends upon this strategic mineral. The original patent and technical research records from which "Polectron" Synthetic was developed became a military secret when the control of General Aniline, formerly enemy-owned, was invested in the Alien Property Custodian. Facilities of the Company were immediately directed toward increased war production and an expanded program of research and development was instituted by the new board of managing directors under the chairmanship of Robert E. McConnell. One of the first

war projects undertaken was the industrial production of a synthetic replacement material for high grade mica to reduce the drain from our supply on hand. High quality mica, such as that used in radio condensers, is available in several fields in the United States but its preparation for use presents many problems. Cost and unavoidable waste are two contributing factors. The mica used in most electronic equipment must be physically pure, free from stains and other inclusions. The mica sheets must also be standard and uniform in thickness and tested for a variation of 1/10,000 of an inch. They must resist heat, moisture and oil and have the toughness to become the very heart of condensers. These were the properties General Aniline had to invest in their basic substance for mica replacement material. That "Polectron" Synthetic has proven its ability to release much high grade mica for other uses is evidenced by priorities ordering its immediate production. Its unique properties of heat resistance and electrical characteristics, essential in mica replacement, are also of basic significance in a wide range of other electronic applications.

R. H. O.

Construction's Magic Rings.—(*Compressed Air Magazine*, Vol. 48, No. 5.) From the Office of War Information we learn that the judicious use of timber has conserved as much as 200,000 tons of structural steel in a single military construction program, and that all in all more than 400,000 tons of steel was saved in this manner during 1942. Much of the credit for this impressive conservation can be given to the metal timber connector—pressed steel rings or malleable iron plates ranging from 2½ to 6 inches in diameter. When placed in circular grooves between adjacent faces of overlapping timbers, they provide larger supporting areas than those obtained by other joining methods. By contrast relatively small supporting areas are furnished by bolts, which are customarily used for this purpose. The addition of a timber connector makes it possible to distribute the load more equally over the cross section of the wood and thus utilize more fully the structural strength of the timber itself. Under the Government war program, numerous towers, bridges, hangars, warehouses, and other structures have been erected of wood by means of timber connectors with astonishing results. For example, wood towers built before the development of the new technique were limited to a height of 80 feet—the height being restricted because of the weakness of the supporting joints. But now, structures of this kind, meeting all engineering requirements and specifications, sometimes reach an elevation of 300 feet. Moreover, the savings in critical metal are often startling, even in

cases of individual buildings. For instance, the use of timber connectors in a blimp hangar 1000 feet long, 153 feet high and having a roof span of 237 feet, enabled the Navy to construct it of wood and thus save about 2,050 tons of structural steel.

R. H. O.

Supplemental Soy.—(*Industrial Bulletin of Arthur D. Little, Inc. No. 189.*) Partially because of its strong individuality in flavor, the soybean is more often heard of than seen on the dinner table. In general, mixtures or blends containing enough soy to be valuable, and yet not flavor-dominant, may be best. With this principle of restraint in mind, greater things may be expected of soy in human food, and the rather pleasant flavor of properly blended soy products may eventually become widely popular. The enrichment of wheat flour by the addition of vitamins and minerals was a forward step, yet enriched flour still lacks much in nutritive value because of its low grade protein content; even whole wheat flour is incomplete food, since wheat protein is deficient in lysine content and is weak in several other amino acids. The soybean, however, is phenomenally rich in lysine, containing some nine per cent., or double the quantity present in the notably good proteins of peanuts and cottonseed. Comparing a soybean patent flour with the straight patent flour, a government nutrition laboratory has reported marked increase in growth promoting value, in rats, for five per cent. content of soybean flour and still greater increase for ten per cent. content, but practically no additional nutritive gain was found for still higher proportions of the soy. Soy concentrations of five per cent. cannot be detected by taste. In war or peace times, the use of soy flour for the protein enrichment of bread is a great nutritional gain and promises to provide a large and steady outlet for soy, particularly since the vitamins in bread are better assimilated when adequate protein is available in the diet. Through the centuries, Asiatic races have gradually developed better and better strains of soy plants as well as new ways of processing the beans. Some strains are of sturdy field type, well suited for the industrial production of soybean oil, defatted grits, flour and plastics, whereas other more tender and delicate strains are grown in gardens and eaten fresh or dried. The cooking of "edible" soybeans well illustrates the heat sensitivity of soybean flavor. When undercooked, the taste is objectionably grass-like, but on longer boiling this vanishes and there appears a delicious flavor resembling roasted chestnuts.

R. H. O.

Rectifying Columns Packed with Glass Fibers Speed Up Production of Ethyl Alcohol.—A marked speed-up in the rate of distillation of the rectifying columns used by the beverage distilling industry to produce 190 proof ethyl alcohol required for explosives, synthetic rubber and other uses, has been made possible by the development by Owens-Corning Fiberglas Corporation of a new glass fiber packing material for the columns. The glass fibers can replace both the tinned-copper bubble plates with which the industry has equipped its columns, and the burned-clay Raschig rings which the industry resorted to as a substitute when tinned copper became unavailable. It is believed that the material will also prove practical for use in distillation applications in the chemical, petroleum and other industries. A principal factor in determining the rate of distillation of an alcohol rectifying column is the amount of exposed surface area which is presented for the condensation of water and other liquids. The ability of the glass fibers to increase the rate of distillation is due to the great increase in exposed surface area presented by the fibers, as compared to the exposed surface area presented by either the bubble plates or the Raschig rings. One method employed in packing the columns with glass fibers consists of placing them in large, expanded metal baskets which fit, one over the other, into the inside of the column. When used at their normal density of 3.5 pounds to the cubic foot, the fibers present 135 square feet of exposed surface area per cubic foot. This compares with an exposed surface area of 56 square feet per cubic foot when the Raschig rings are used. The beverage distilling industry formerly employed a column with a copper shell tinned on the inside, but the columns it is now erecting are built like a silo, of clay tile, cypress staves, or steel plates salvaged from tanks formerly used for other purposes. The new columns are from 4 to 10 feet in diameter, and are approximately 50 feet high. The operation of columns packed with glass fibers is identical with that of columns packed with Raschig rings, or fitted with the tinned-copper bubble plates. Heated vapors from stills producing the normal run of 120–140 proof alcohol pass up through the column. Water and other liquids with a boiling point higher than the 170 degrees, Fahrenheit, boiling point of ethyl alcohol condense on the bubble plates, Raschig rings or glass fibers, and flow back to be reheated and re-vaporized by the rising vapors from the still, until the last vestige of alcohol is extracted from them. The vapors which finally pass out through the top of the column into a condenser become 190 proof alcohol with only 5 per cent. of water content.

R. H. O.

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Vol. 236 **SEPTEMBER, 1943** **No. 3**

CONTENTS

Henson—Inventor of the Airplane ..... A. F. ZAHM 235

Otto von Guericke: A Neglected Genius ..... THOMAS COULSON 241

The First Governmental Application of Photography ..... DON D. NIBBELINK 265

On the Forced Vibrations of Non-Linear Springs ..... C. R. WYLIE, JR. 273

Some Preliminary Investigations on Dehydration of Fruits and Vegetables with Infrared Energy ..... JOHN E. NICHOLAS 285

On the Geometry of Colorspace ..... DOMINA EBERLE SPENCER 293

Notes from the National Bureau of Standards 303

The Franklin Institute Notes 313

Notes from the Biochemical Research Foundation 316

Book Reviews 321

Publications Received 324

Current Topics 240, 272, 312, 325

Published by

THE FRANKLIN INSTITUTE OF THE STATE OF PENNSYLVANIA
Prince and Lemon Streets, Lancaster, Penna., and
Benjamin Franklin Parkway at Twentieth St., Philadelphia, Penna.

SIX DOLLARS PER YEAR (Foreign Postage Additional) **SINGLE NUMBERS, SIXTY CENTS**

Indexes to the semi-annual volumes of the JOURNAL are published with the June and December numbers. The contents are also indexed in *The Industrial Arts Index.*

1874

Riehlé Brothers, Philadelphia, introduce a new testing machine

At the 27th Exhibition of American Manufactures held by The Franklin Institute in 1874, two samples of rubber belting were shown.

An alert committee of the Institute conceived the idea of testing these samples of rubber belting on the Riehlé Brothers' testing machine.

One sample of four ply three inch wide rubber belting broke (pulled apart) at 3500 pounds. In a second test it broke at 3000 pounds. The makers of this sample were awarded a Certificate of Honorable Mention.

The Franklin Institute took the "guess" out of Rubber belting, and put "specifications" in its place. From that time forward rubber belting was built to specification. A purchaser could know exactly what to expect in belting, based on scientific tests for strength.

Scientific tests are constantly made in the development and manufacture of all Lee Rubber & Tire Corporation products. Specifications are not approximated. They are exceeded. Have you seen the rubber exhibit in the Chemistry Section?